ZBIRKA IZBRANIH POGLAVIJ IZ FIZIKE

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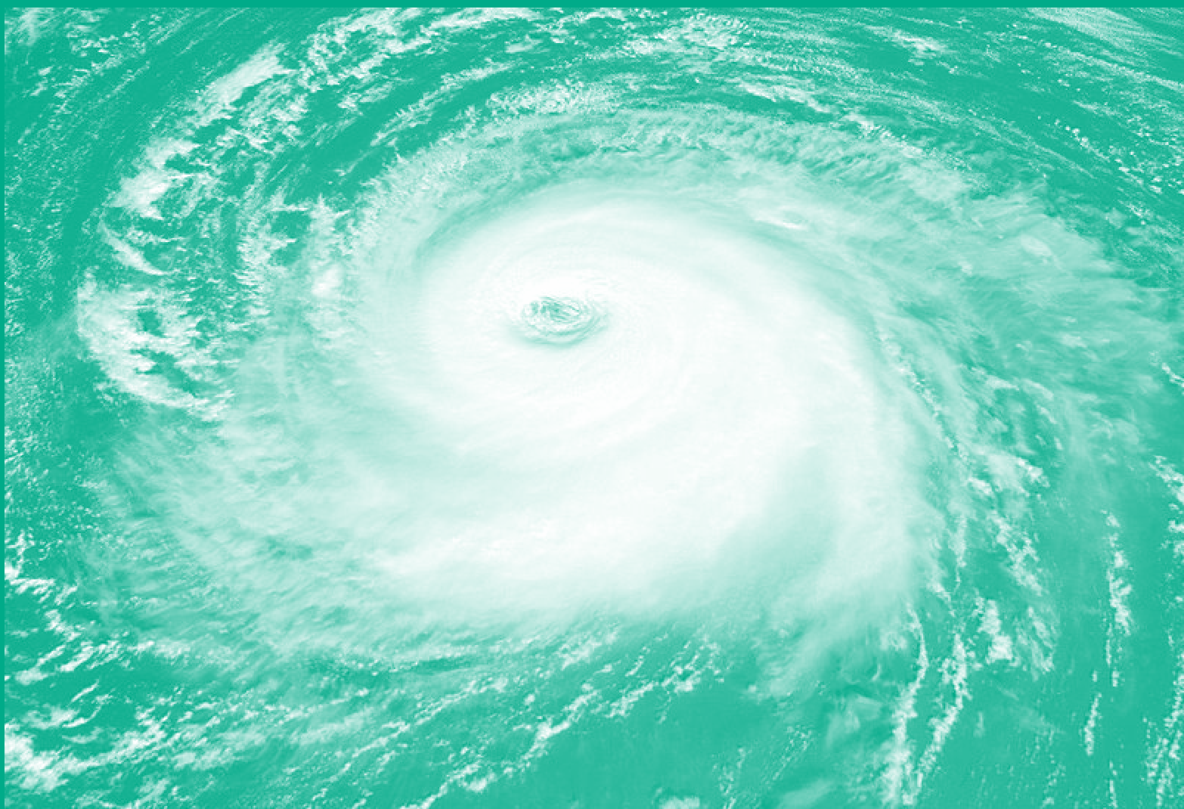
Saša Gaberšek
Gregor Skok
Rahela Žabkar

INTRODUCTION TO METEOROLOGY: SOLVED PROBLEMS



DMFA – založništvo

Saša Gaberšek, Gregor Skok
and Rahela Žabkar

INTRODUCTION TO METEOROLOGY: SOLVED PROBLEMS

DMFA – založništvo
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Contents

Preface

This booklet is an English translation of the Slovenian language booklet *Rešene naloge iz Osnov meteorologije* (ISBN: 978-961-212-199-0). It is intended as a supplement to the *Introduction to Meteorology* course at the Faculty of Mathematics and Physics, University of Ljubljana. An integral part of this course is problem-solving exercises. The booklet is based on the problems that were initially collected by Prof. Zdravko Petkovšek, Prof. Jože Rakovec, Asst. Prof. Tomaž Vrhovec, Neva Pristov, MS, and Gregor Gregorič, PhD. The initial set of problems was a core that was later extended by the authors. Each problem in the booklet is presented with solution.

The booklet is organized into sections related to the book *Osnove meteorologije za naravoslovce in tehnike* (ISBN: 978-961-212-111-2), written by Prof. Jože Rakovec and Asst. Prof. Tomaž Vrhovec.

The booklet is dedicated to our colleague Tomaž Vrhovec, who lost his life in an avalanche in the Julian Alps.

The authors also want to thank Prof. Jože Rakovec for reviewing and providing useful advice and to the student Žiga Valentič for help in finding errors in the original Slovenian language version of the booklet.

The authors also want to thank Veronika Hladnik, MS, for help with the English translation of the booklet.

The Authors

Chapter 1

Units

1.1 Convert the following units for pressure:

1000 mbar = Pa,

600 mbar = hPa,

500 mbar = bar.

1.2 Convert the following units for temperature:

12.5 °C = K,

290 K = °C,

−14 °C = K.

1.3 Convert from one set of units to another:

3 days = seconds,

20000 h = years,

900 m^2 = km^2,

3 litres = m^3,

30 m/s = km/h,

100 km/h = m/s,

100 MW = W,

1400 W/m^2 = W/km^2,

0.005 K/s = °C/hour,

1.5 mbar/100 km = Pa/m.

100 MJ = kWh,

5 kWh = MJ.

Chapter 2

Structure and layers of the atmosphere

2.1 Calculate the molar mass of the air, assuming that the mass fraction of oxygen in the atmosphere is 24% and nitrogen 76%.

Solution:

$$M_{O_2} = 32 \text{ kg/kmol}, \quad M_{N_2} = 28 \text{ kg/kmol}.$$

$$p = \frac{mR^*T}{VM} = p_{O_2} + p_{N_2} = \frac{m_{O_2}R^*T}{VM_{O_2}} + \frac{m_{N_2}R^*T}{VM_{N_2}}.$$

The mass fractions: $m_{O_2} = m \cdot 0.24, \quad m_{N_2} = m \cdot 0.76.$

From here we obtain: $\frac{1}{M} = \frac{0.24}{M_{O_2}} + \frac{0.76}{M_{N_2}}$,

$$M = 28.866 \text{ kg/kmol}.$$

2.2 What is the mass of the air in a classroom with dimensions 10 m × 10 m × 3 m, if the atmospheric pressure is 1013 mbar and the temperature 25 °C?

2.3 What is the molar mass of moist air that has the partial water vapour pressure of 15 mbar and total atmospheric pressure (the sum of dry air and water vapour) of 1010 mbar?

2.4 What is the density of dry air, if the atmospheric pressure is 1000 mbar and the temperature 30 °C (−16 °C)?

2.5 How much does the air density change if we descend from an altitude of 1500 metres, where the temperature is 20 °C and the atmospheric pressure is 855 mbar to an altitude of 450 m, where the pressure is 960 mbar and the temperature 30 °C?

2.6 What is the mass of argon in the room in which the temperature is 15 °C, the atmospheric pressure 1000 mbar, the humidity 30%, and the room volume 100 m^3? The mass fraction of argon is 1.28%.

2.7 Estimate the mass of the atmosphere of the Earth.

Chapter 3

Hydrostatics

3.1 At sea level the measured atmospheric pressure was 1005 mbar and the temperature was 15 °C. Calculate the pressure at the altitude of 4000 m for the following two cases:

a) atmosphere is isothermal (temperature does not changes with height),

b) Temperature is linearly decreasing with height at a rate of 5 K/km.

Solution:
a) Because the atmosphere is isothermal, we can use the equation:

$$p = p_0 \cdot \mathrm{e}^{-\frac{g(z-z_0)}{RT}},$$

where p_0 is the atmospheric pressure on the sea level – 1005 mbar and z_0 the altitude of the sea level – 0 m.

$$p = 625 \text{ mbar}.$$

b) Because the temperature is linearly changing with height, we can use equation:

$$p = p_0 \cdot \left[1 + \left(\frac{\partial T}{\partial z}\right) \frac{z - z_0}{T_0}\right]^{-\frac{g}{R\left(\frac{\partial T}{\partial z}\right)}},$$

where for $\left(\frac{\partial T}{\partial z}\right)$ we insert -0.005 K/m, because the temperature is decreasing with height. p_0, z_0 and T_0 are atmospheric pressure, altitude (0 m) and temperature at the sea level.

$$p = 614 \text{ mbar}.$$

3.2 For the examples below, calculate the height at which the atmospheric pressure equals 10 mbar. For all examples, take the default assumption that the atmospheric pressure and the temperature at sea level are 1013 mbar and 273 K.

a) Homogeneous atmosphere (density of the air is constant with height).

b) Isothermal atmosphere.

c) Atmosphere in which the temperature linearly decreases with height at a rate of 6.5 K/km.

Solution: a)

$$\Delta z = \frac{\Delta p}{\rho g} = 7908 \text{ m},$$

b)

$$\Delta z = \frac{RT}{g} \ln \frac{p_0}{p} = 36884 \text{ m},$$

c)

$$\Delta z = \frac{T_0}{\left(\frac{\partial T}{\partial z}\right)} \left[\left(\frac{p_1}{p_0}\right)^{-\frac{R\left(\frac{\partial T}{\partial z}\right)}{g}} - 1\right] = 24548 \text{ m}.$$

3.3 Calculate the atmospheric pressure at the following altitudes: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 km in the atmosphere, if the temperature decreases with height linearly at a rate of −0.01 K/m and the temperature at the ground is 10 °C?

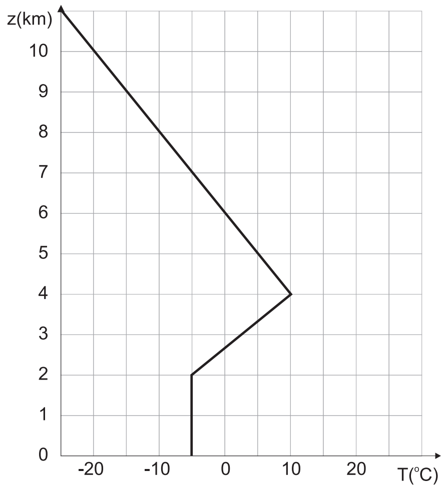
3.4 Calculate the height of 10 mbar level for the atmosphere with the temperature and atmospheric pressure at the ground at 15 °C and 1013 mbar. Between altitudes of 0 m and 100 m, the temperature increases by 2 K, between 100 m and 1000 m the temperature decreases by 7 K, above this height to the altitude of 3000 m the temperature is constant and from there upward the temperature decreases for 6.5 K/km up to the tropopause at the height of 12 km. From there upward, the temperature is constant.

3.5 At a height of 1500 m, the atmospheric pressure is 850 mbar. Calculate the pressure at the ground for the following two cases:

a) The temperature from the ground upward is decreasing at 5 K/km and reaches −7 °C at the height of 1500 m,

b) Up to a height of 300 m above the ground lies cold air with the temperature of −1 °C. Above this height, the temperature rapidly (discontinuously) increases to 4 °C and then decreases with height by 8 K/km.

3.6 A meteorological balloon measured a vertical profile of temperature that is shown on the graph below. Independently, a meteorological station at an altitude of 2 km measures the temperature at −5 °C and the atmospheric pressure 850 mbar.



a) What is the atmospheric pressure at sea level?

b) What is the atmospheric pressure at the height of 4 km? What is the air density at that height?

c) What is the atmospheric pressure at the height of 9 km?

d) At which altitude will the atmospheric pressure be 850 mbar, if conditions of standard atmosphere are assumed?

3.7 Determine the thickness of the air layer, if the atmospheric pressure at the lower boundary is 779 mbar and at the upper boundary 545 mbar,

a) if the layer is isothermal with the temperature 273 K,

b) if the temperature at the lower boundary is 273 K and at the upper boundary 266 K. Assume that the vertical temperature change is linear.

3.8 Calculate the height at which the atmospheric pressure is equal to 700 mbar. The atmospheric pressure at the ground is 1000 mbar and the ground is located 207 m above the sea level. In the intermediate layer, the temperature decreases linearly with height with the gradient of 6.5 K/km and the temperature at the ground is 22 °C.

3.9 Calculate the height at which the atmospheric pressure is equal to 100 mbar if standard atmospheric conditions are assumed?

3.10 The aeroplane uses an altimeter, which works on the basis of atmospheric pressure measurements. The altimeter assumes conditions of ICAO standard atmosphere (p_0 = 1013 mbar, T_0 = 15 °C, $\left(\frac{\partial T}{\partial z}\right) = -6.5$ K/km) and shows an altitude of 9000 m. What is the real altitude of the plane above the region, if the atmospheric pressure at sea level is 980 mbar, the temperature 0 °C and the temperature decreases with height by 8 K/km?

3.11 When a mountaineer went from Aljažev dom in Vrata (the altitude of 1015 m), where the atmospheric pressure was 880 mbar, he set his altimeter and looked at the thermometer, which showed 15 °C. When he came to Kredarica, the temperature was 5 °C and a meteorological observer told him that the temperature was essentially constant all day. His altimeter was showing 2500 m. By how much did the atmospheric pressure change on Kredarica, which is at the altitude of 2515 m?

3.12 How thick and at which altitude is the air layer between 900 mbar and 800 mbar in the atmosphere. Atmospheric pressure and the temperature at the sea level are 1000 mbar and 10 °C and from there upward the temperature is decreasing by 5 K/1 km?

3.13 A plane is circling above Portorož. The temperature and atmospheric pressure are 5 °C and 1020 mbar. A zero value of the altimeter, which is assuming a ICAO standard atmosphere, is set to this pressure and the altimeter shows that the plane is at the altitude of 3000 m. Because the bora wind is blowing, the air in the lower layer of the troposphere is well mixed, so that the temperature is decreasing with the altitude by 9 K/km. What is the true altitude of the plane?

3.14 During the night, the layer of air between 1000 mbar and 950 mbar losses 1 MJ/m^2 of energy due to emitted radiation. How much do the temperature and the thickness of the layer decrease during the night?

3.15 The layer at the ground, between 1013 mbar and 950 mbar, absorbs 6 kWh of solar radiation energy on each square metre. What is the difference between the original and new thickness of this layer?

3.16 What is the difference in height where the atmospheric pressure equals 925 mbar, at Ljubljana and at Koper? The ground level atmospheric pressure and temperature at Koper (which is at sea level) are 1009 mbar and 14 °C while at Ljubljana (300 m above sea level) the temperature is the same as at Koper while the atmospheric pressure is 975 mbar. Assume a temperature decrease with height of −6.5 K/km.

3.17 What is the atmospheric pressure, if the height of the column of mercury on the barometer is 732.5 mm and the temperature 15 °C? The density of mercury is 13.5 kg/l.

3.18 How large is the error when performing the atmospheric pressure reduction at a meteorological measuring station that is 300 m above the sea level, if the measured temperature is 288 K and the actual temperature between the station and sea level is higher by 1 K?

3.19 In Slovenia the atmospheric pressure reduction uses the temperature measured at the meteorological station. For the stations at the high altitudes, that kind of methodology leads to major errors. This is the reason that high-altitude stations use a different methodology for reduction. Thus, on Kredarica (altitude 2515 m) they determine the height of the 700-mbar isobar. What was the true measured atmospheric pressure on Kredarica, if the temperature was 4 °C and the 700-mbar level was calculated to be at the height of 3000 m?

3.20 How is the temperature changing with height in a homogeneous atmosphere?

3.21 During the winter, the formation of a cold-air lake in basins is a common phenomenon. The temperature at the ground is -5 °C, and in the lower 300 m of the atmosphere an inversion exists with a vertical temperature increase of 0.001 K/m. Above this altitude, the vertical temperature gradient is the same as in the standard atmosphere. How much influence does the inversion have on the calculation of atmospheric pressure reduction? Assume that the bottom of the basin is 300 m above the sea level.

3.22 What is the geopotential of 500 mbar level in the standard atmosphere?

Chapter 4

Basic laws

4.1 Calculate the size of the horizontal component of the Coriolis acceleration at different latitudes (10°, 30°, 50°, 80°) for the movement with speed 10 m/s.

Solution:
Horizontal component of the Coriolis acceleration is written as fv, where v is the wind speed and f the Coriolis parameter. $f = 2\Omega \sin\varphi$, where Ω is the angular velocity of the rotation of the Earth and φ the latitude.

$$\begin{aligned} fv(\varphi = 10^\circ) &= 2.52 \cdot 10^{-4} \text{ m/s}^2, \\ fv(\varphi = 30^\circ) &= 7.27 \cdot 10^{-4} \text{ m/s}^2, \\ fv(\varphi = 50^\circ) &= 1.11 \cdot 10^{-3} \text{ m/s}^2, \\ fv(\varphi = 80^\circ) &= 1.43 \cdot 10^{-3} \text{ m/s}^2. \end{aligned}$$

4.2 At what circling radius is the centrifugal force equal to 10% of the Coriolis force? Assume a latitude of 30° N and velocity 15 m/s.

4.3 The air is moving horizontally towards the northeast at the speed of 25 m/s. To which direction does the horizontal component of the Coriolis force point and how big is the force at 60° N?

4.4 How much lighter does the mountaineer feel, if he climbs on Kilimanjaro, which is 5800 m high and is located at 2° N? Otherwise, the climber lives at the altitude of 300 m at 50° N?

4.5 How much less weight is displayed on a scale, if a man is weighed on a train that is travelling through Slovenia at the speed of 120 m/s towards the west? And how much if the train is travelling at the same speed along the equator?

4.6 What is the ratio between the size of vertical and horizontal components of the pressure gradient force at the ground, if the atmosphere is standard and the atmospheric pressure changes in the horizontal direction by 3 mbar at the distance of 180 km?

4.7 What should be the volume of the hot air balloon, in which the temperature of the air is 50 °C, to raise itself and the basket with a combined weight of 300 kg? The temperature of the surrounding air is 10 °C and the atmospheric pressure 1020 mbar.

4.8 The two maps below show the height of the 700 mbar and 850 mbar levels. For the three shown points (A, B and C) calculate:

a) the size of the specific horizontal pressure gradient force and compare it with the vertical pressure gradient force.

b) specific centrifugal force.

Meridians are spaced apart by 20° and parallels by 10°. The temperatures are known in the three points:

p (mbar)	T_A (K)	T_B (K)	T_C (K)
700	273	272	255
850	278	290	262

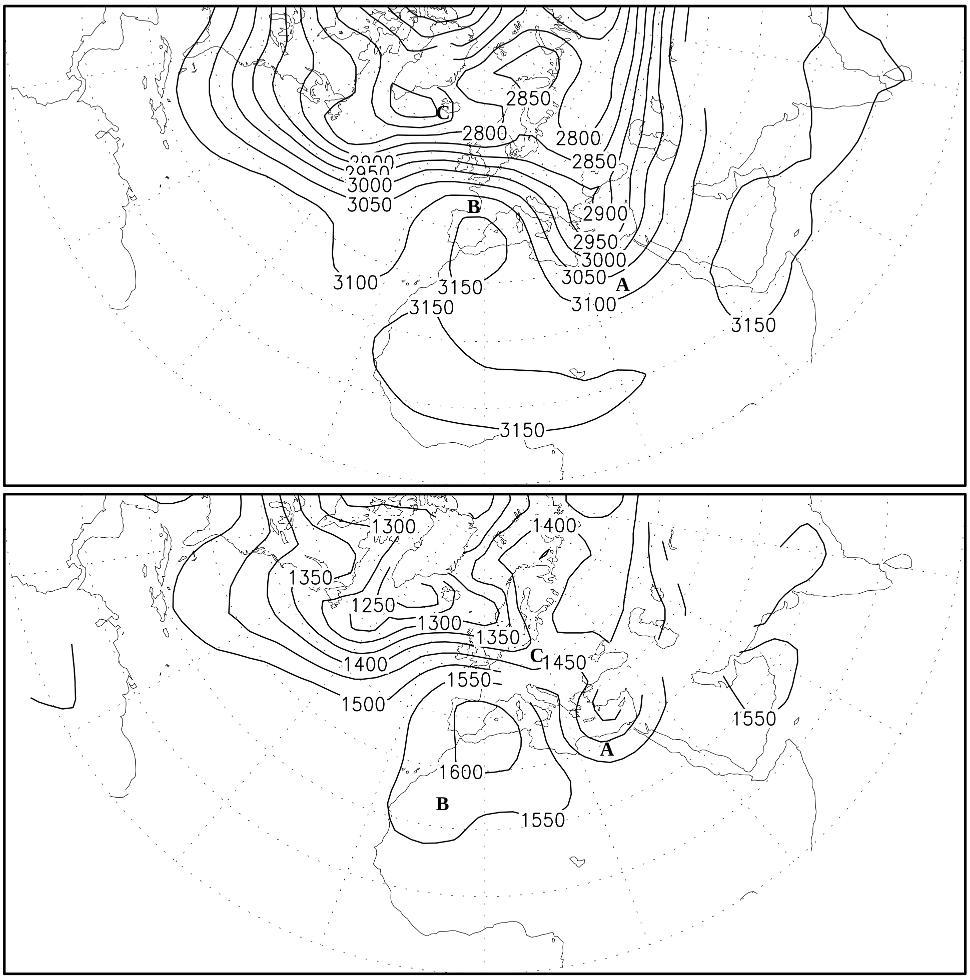


Figure 4.1: The height of the 700-mbar (top) 850-mbar levels in metres.

Solution:
First, calculate the air density:

p (mbar)	ρ_A (kg/m^3)	ρ_B (kg/m^3)	ρ_C (kg/m^3)
700	0.9	0.8	0.96
850	1.06	1.02	1.13

Calculate the specific horizontal pressure gradient force using the distance between the neighbouring contour lines of the height of the constant atmospheric pressure:

$$-\frac{1}{\rho}\frac{\partial p}{\partial n} = -\frac{\partial \Phi}{\partial n}.$$

Calculate vertical component using the hydrostatic equation:

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g.$$

Calculate specific centrifugal force indirectly via the gradient wind.

a) 700 mbar

point	R (km)	f (s^{-1})	V (m/s)	$-\frac{1}{\rho}\frac{\partial p}{\partial n}$ (m/s^2)	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$ (m/s^2)	$\frac{V^2}{R}$
A	1000	$8.3 \cdot 10^{-5}$	12.8	$1.22 \cdot 10^{-3}$	9.81	$1.6 \cdot 10^{-4}$
B	−900	$1.0 \cdot 10^{-4}$	20.7	$1.63 \cdot 10^{-3}$	9.81	$4.8 \cdot 10^{-4}$
C	500	$1.3 \cdot 10^{-4}$	8.4	$1.22 \cdot 10^{-3}$	9.81	$1.4 \cdot 10^{-4}$

b) 850 mbar

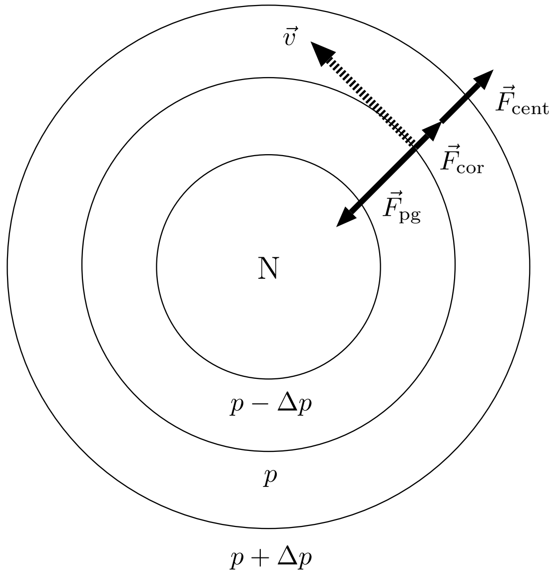
point	R (km)	f (s^{-1})	V (m/s)	$-\frac{1}{\rho}\frac{\partial p}{\partial n}$ (m/s^2)	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$ (m/s^2)	$\frac{V^2}{R}$
A	1000	$7.3 \cdot 10^{-5}$	14.1	$1.22 \cdot 10^{-3}$	9.81	$2.0 \cdot 10^{-4}$
B	−1000	$6.2 \cdot 10^{-5}$	6.9	$3.7 \cdot 10^{-4}$	9.81	$4.8 \cdot 10^{-5}$
C	500	$1.2 \cdot 10^{-4}$	7.5	$9.8 \cdot 10^{-4}$	9.81	$1.1 \cdot 10^{-4}$

Chapter 5

Steady state horizontal winds

5.1 For a circular area of low pressure, draw the equilibrium of the forces and the velocities that is valid over the ocean (while neglecting the friction). Write the equation for determining the wind speed.

Solution:



From the equilibrium of the three forces, the following equation is obtained:

$$\vec{F}_{\rm cent} + \vec{F}_{\rm cor} + \vec{F}_{\rm pg} = 0,$$
$$\frac{v^2}{R} + fv - \frac{1}{\rho}\,|\nabla p| = 0.$$

v can be expressed via a solution to a quadratic equation as (only the positive solution is physically meaningful):

$$v = \frac{1}{2}\left(-fR + \sqrt{f^2R^2 + 4\,\frac{R}{\rho}\,|\nabla p|}\right) .$$

5.2 By what angle does the wind deviate away from the isobars in a circular area of low atmospheric pressure, if the friction is not neglected? The friction coefficient is 0.00001 s^{-1}, the radial component of the pressure gradient is 2 mbar/100 km and the air density 1 kg/m^3. The cyclone is located at 45° N. What is the wind speed at a distance of 300 km from the centre?

Solution:
Due to friction, the wind in the cyclone deviates towards the centre. If we equalize the force components in the two directions, we obtain:

$$fv + \frac{v^2}{R} = \frac{1}{\rho}\,|\nabla p|\cos\beta,$$
$$kv = \frac{1}{\rho}\,|\nabla p|\sin\beta.$$

Assuming that β is small: $\cos\beta \approx 1$ and $\sin\beta \approx \beta$. From the first equation, the wind velocity can be expressed, which is the same as for the example without friction. The velocity is 13.52 m/s. From the second equation, the deviation angle can be expressed:

$$\beta = \frac{kv\rho}{|\nabla p|} = 0.068 \text{ radian} = 4°.$$

5.3 In a cyclone, at what radius does the wind equals to 8 m/s, if the gradient of the atmospheric pressure in the radial direction is 1 mbar/100 km. The friction can be neglected and the air density is 1 kg/m^3.

5.4 Calculate the wind speed in the anticyclone at 45° N, if the gradient of the atmospheric pressure is 2 mbar/400 km and the radius of the curvature of isobars is 400 km. Assume air density of 0.7 kg/m^3.

5.5 In the anticyclone, the air circulates at 45° N at radius of 1000 km. At the height at which the atmospheric pressure equals 500 mbar the wind speed is 20 m/s. What is the size of the horizontal atmospheric pressure gradient that determines this movement?

5.6 How strong is the geostrophic wind at 30° N if the size of the atmospheric pressure gradient is 2 mbar/100 km and the air density 0.5 kg/m^3?

Solution:
The geostrophic equilibrium applies when the pressure gradient force is balanced with the Coriolis force:

$$fv_{\mathrm{g}} = \frac{1}{\rho}\,|\nabla p|\,,$$
$$v_{\mathrm{g}} = \frac{1}{\rho f}\,|\nabla p| = 55 \text{ m/s}.$$

5.7 In the field of the straight isobars at our latitudes (46° N, 15° E) the wind is blowing at the speed of 30 km/h and deviates by 15° from the direction of the isobars. Under the influence of which forces does the wind blow? What are the sizes of these forces? How large is the gradient of the atmospheric pressure? The air density is 0.5 kg/m^3.

5.8 Calculate the wind speed in a cyclone at 70° N, if the atmospheric pressure gradient size is 4 mbar/450 km and the radius of the isobar curvature is 400 km. The air density is 0.7 kg/m^3.

5.9 What is the ratio of the speeds between the gradient and the geostrophic wind at 50° N, if the atmospheric pressure gradient size is 1 mbar/100 km and the radius is 1500 km? The air density is 1 kg/m^3.

5.10 On a weather map in a scale 1 : 5000000 at 60° N, the distance between the two nearby straight isobars is 1 cm. The atmospheric pressure interval between the isobars is 4 mbar. What atmospheric pressure gradient size and what geostrophic wind speed corresponds to this field? What is the wind speed at the same interval between the isobars at 50° N? The wind direction in this example is not relevant. The air density is 1 kg/m^3.

5.11 In the field of the straight isobars, the wind is blowing at 60° N over an extensive plane. The atmospheric pressure gradient size is 2 mbar/150 km and the ground is uniformly rough, such that the coefficient of the linear friction is 10^{-4} s^{-1}. Draw the balance of the forces, specify the size of the specific forces, and calculate the wind speed. The air density is 1 kg/m^3.

5.12 Due to friction, the wind deviates by 30° to the left of the straight isobars. The isobars are plotted every 5 mbar and the distance between two nearby isobars is 200 km. The coefficient of the linear friction is 10^{-4} s^{-1}. What is the wind speed and how large are the forces, that hold the balance at that wind? The air density is 0.7 kg/m^3.

5.13 What is the difference in atmospheric pressure between the point on the edge and in the centre of the tropical cyclone (hurricane), if on the edge, 500 km from the centre, the wind is blowing at a speed of 200 km/h and the atmospheric pressure is decreasing towards the centre linearly? What are the pressure differences, if you assume that the frictional force exists that is proportional to the square of the speed? The coefficient of the square friction is 10^{-7} m^{-1}. The air density is in both cases 1 kg/m^3.

5.14 What is the atmospheric pressure at the centre of the tropical cyclone, if on its edge, 400 km from the centre, the wind blows at the speed of 150 km/h and the atmospheric pressure is 970 mbar? The pressure field in a hurricane is parabolic with the minimum in the centre. The air density is 1 kg/m^3.

5.15 The undulating westerly wind is blowing at the height of 5500 m around the Earth. At 30° N there is a trough, where the air circulates through the part of the circle with the radius of 1000 km and the wind blows at the speed of 20 m/s. What is the radius of the rotation at the ridge, at 60° N, if the speed is the same there?

5.16 A tornado is rigidly rotating (the angular speed is independent of the radius). What is the atmospheric pressure field around the tornado centre, if the air density is the same everywhere (horizontal and vertical homogeneity)?

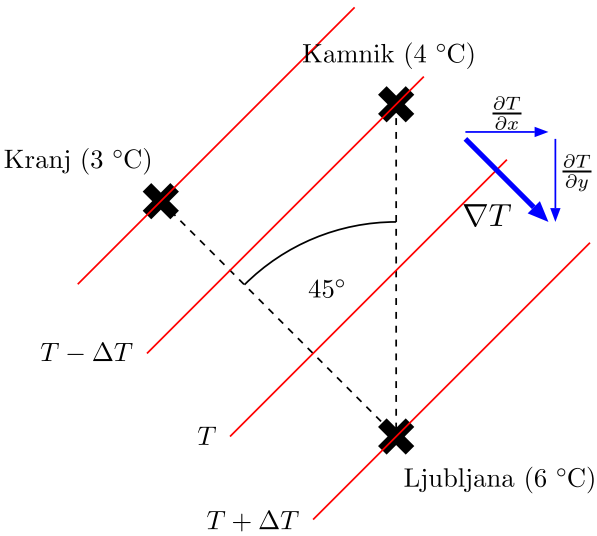
5.17 A tornado axis is tilted by 30 degrees from the vertical. What should be the speed of the rotation 100 m from the axis of the rotation, if the atmospheric pressure is not changing with the height?

Chapter 6

Local, individual and advective changes

6.1 The temperature in Ljubljana is 6 °C, while in Kranj, which lies 20 km northwest of Ljubljana, the temperature is 3 °C. In Kamnik, which lies 20 km north of Ljubljana, the temperature is 4 °C. The atmosphere is calm, the temperature field is changing linearly everywhere (the temperature field can be written with the equation $T = T_0 + ax + by$). In which direction does the temperature gradient point and what is its value? Draw the sketch of the temperature field with the isotherms.

Solution:



In order to calculate the temperature gradient the constants T_0, a and b need to be determined first. The three known temperatures can be inserted into the equation of the temperature field ($T = T_0 + ax + by$) which produces three linear equations for the three unknown variables.

If the origin of the coordinate system ($x = 0$, $y = 0$) is placed at Ljubljana and temperature ($T_{LJ} = 6$ °C) inserted into the equation, the $T_0 = 6$ °C is obtained. If the data for Kamnik is inserted ($T_{KM} = 4$ °C, $x = 0$, $y = 20$ km) the $b = -0.10$ K/km is obtained. Finally, from the data for Kranj ($T_{KR} = 3$ °C, $x = -20$ km $\cos 45°$, $y = 20$ km $\sin 45°$) the $a = 0.11$ K/km is obtained.

The temperature gradient is

$$\nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}\right) = (a, b) = (0.11 \text{ K/km}, -0.1 \text{ K/km}).$$

6.2 The field of the atmospheric pressure can be described with the function

$$p = p_0 + a \cdot y,$$

where $p_0 = 1000$ mbar and $a = -2$ mbar/100 km. Calculate the atmospheric pressure gradient. Draw the pressure field using the isobars and draw into this field the vectors of the gradient. What is the pressure at a point, that has the coordinates $x = 100$ km, $y = 200$ km?

6.3 Over the ocean, the atmospheric pressure field can be described with the following a function:

$$p = p_0 + ay + bx^2,$$

where $p_0 = 1000$ mbar, $a = -2$ mbar/100 km and $b = 3 \cdot 10^{-4}$ mbar/km^2. Calculate the pressure gradient. Draw the pressure field using the isobars, and draw on this field the vectors of the gradient. What is the pressure at the point with the coordinates $x = 100$ km, $y = 100$ km?

6.4 Over the ocean, the atmospheric pressure field can be described with the following a function:

$$p = p_0 + a(y - y_0) + b(x - x_0),$$

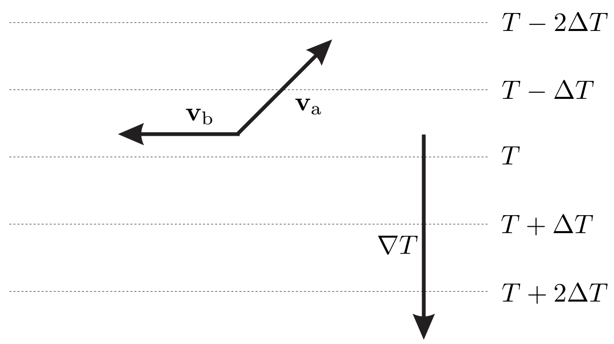
where $x_0 = 500$ km, $y_0 = 300$ km, $a = 1$ mbar/100 km, $b = 0.5$ mbar/100 km and $p_0 = 1010$ mbar. Draw the pressure field with the interval between 2-mbar isobars. Calculate the pressure gradient. Draw the vector of the pressure gradient on the atmospheric pressure field. Calculate the atmospheric pressure at the point with the coordinates $x = 300$ km, $y = -100$ km?

6.5 Over Slovenia, the temperature decreases from south to north at a rate of 3 K/100 km. How much will the temperature change in three hours, if

a) there is a south-westerly wind with speed of 10 m/s,

b) there is an easterly wind with speed of 10 m/s,

Solution:
The temperature field, the temperature gradient (∇T) and the wind vectors ($\mathbf{v_a}$ and $\mathbf{v_b}$) are shown in the sketch below.



The temperature gradient always points in the direction of the maximum increase of temperature and, in this case, only has the component in the y direction. It can be written as $\nabla T = (0,\ -3 \text{ K/100 km})$.

Similarly, the wind vector for case a can be determined:

$$\mathbf{v_a} = (10 \cdot \cos 45^\circ \text{ m/s},\ 10 \cdot \sin 45^\circ \text{ m/s}) = (5\sqrt{2} \text{ m/s},\ 5\sqrt{2} \text{ m/s}).$$

The equation that is connecting the local, individual and advective changes can be used. In this case, the individual change ($\frac{dT}{dt}$) is equal to zero, because the air is not heating or cooling.

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{dT}{dt} - \mathbf{v_a}\nabla T = -\mathbf{v_a}\nabla T \\ &= -(5\sqrt{2}\ \text{m/s},\ 5\sqrt{2}\ \text{m/s}) \cdot (0,\ -3\ \text{K}/100\ \text{km}) \\ &= -5\sqrt{2}\ \text{m/s} \cdot 0 - 5\sqrt{2}\ \text{m/s} \cdot (-3\ \text{K}/100\ \text{km}) \\ &= 0.764\ \text{K/h}.\end{aligned}$$

In three hours, the temperature will change by

$$\Delta T = \frac{\partial T}{\partial t}\Delta t = 0.764\ \text{K/h} \cdot 3\ \text{h} = 2.3\ \text{K}$$

For the second case (the easterly wind) the wind vector can be composed in a similar way $\mathbf{v_b} = (-10, 0)$ m/s. Since the wind and the temperature gradient are perpendicular the $\frac{\partial T}{\partial t}$ will be equal to zero and the temperature will not change.

6.6 Over an area, the temperature is decreasing northwards at the rate of 1 K/100 km and the wind is blowing from a southwest direction at the speed of 10 m/s. How does the temperature change, if there are no individual changes of the temperature? How fast should the wind blow from the south so that the temperature will be changing at the same rate as before?

6.7 How much does the temperature change on the meteorological station, if there is cloudy weather, the south-westerly wind is blowing at the speed of 12 km/h and the temperature in the atmosphere is decreasing from south to north: 100 km towards the south the temperature is 12 °C and 50 km towards the north the temperature is 6 °C?

6.8 A ship is sailing straight from one island to a second island. The first island lies at 14° E and 38° N and the second at 15° E and 39° N. On the ship, it was recorded that the temperature increased for 1 K in three hours. The ship sails at the speed of 12 knots (6 m/s). What is the temperature at the second island, if the temperature is 22 °C at the first island? The temperature between the islands is changing linearly, and the atmosphere is calm.

6.9 Over Slovenia, the temperature increases from the northeast to the southwest at a rate of 5 K/100 km. The air is becoming warmer because of the received energy of the solar radiation and its temperature increases by 3 K/h. What will be the temperature after 2 hours, if now the temperature is 15 °C and the southerly wind is blowing at a speed of 10 m/s?

6.10 Over Europe, the temperature field can be described with the equation $T = T_0(ax^2 + bxy^2 + c)$. The temperature was measured at three locations: $T(x = 20\ \text{km},\ y = 100\ \text{km}) = 10$ °C, $T(x = -200\ \text{km},\ y = 100\ \text{km}) = 5$ °C and $T(x = 100\ \text{km},\ y = -150\ \text{km}) = -7$ °C. What is the rate of temperature change at location $x = 100$ km, $y = 100$ km, if a north-westerly wind is blowing with speed of 15 m/s?

6.11 An aeroplane is flying at the height of 12 km, and its path intersects a medium sized cyclone. The closest distance to the centre of cyclone that the airplane reaches is 1500 km. In the centre of the cyclone, the air is colder while in the surrounding area the air is warmer. When the plane is 2000 km from the centre, it measures the temperature as being −55 °C, when it is closest to the centre, it measures −58 °C. What is the temperature gradient in the radial direction, if we assume, that the cyclone is circularly symmetric?

6.12 Near a big chimney, the measured concentration of sulphur dioxide is 400 μg/m^3. A measuring station is located north from the chimney, 6 km away. First, a light south-westerly wind is blowing at the speed of 3 m/s and it turns into ane equally strong southerly wind. The wind starts to carry the pollutants towards the measuring station. In the direction parallel with the wind the pollutant is distributed uniformly with the gradient 50 μg/m^3 km. After how much time will the measured concentration at the station reach 350 μg/m^3?

Chapter 7

Humidity

7.1 The relative humidity in the room is 70% and the temperature 18 °C. The atmospheric pressure is 1000 mbar. What are the values of a) the vapour pressure, b) the absolute humidity, c) the specific humidity, d) the mixing ratio and e) the dew point temperature?

Solution:

a) To determine the vapour pressure, the saturated vapour pressure first needs to be calculated (it is a function of the temperature only) using the Claussius-Clapeyron equation:

$$\begin{aligned} e_s(T) &= e_{s0}\, \mathrm{e}^{\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T}\right)} \\ &= 20.84 \text{ mbar}, \end{aligned}$$

where $e_{s0} = 6.1$ mbar and $T_0 = 273$ K. h_i and R_v are the latent heat of vaporization and the meteorological gas constant for the water vapour (2.5 MJ/kg and 461 J/kg K).

Since relative humidity is $f = 70\%$, the vapour pressure is 70% of the saturated vapour pressure.

$$e = e_s \cdot f = 14.59 \text{ mbar}.$$

b) The absolute humidity is the density of the water vapour ρ_v for which the ideal gas equation can be used $e = \rho_v R_v T$

$$\rho_v = \frac{e}{R_v T} = 0.0108 \text{ kg/m}^3.$$

c) The specific humidity q is the mass concentration of the water vapour in the air

$$\begin{aligned} q &= \frac{m_v}{m} = \frac{\rho_v}{\rho} = \frac{e}{p}\frac{R}{R_v} \\ &= 9.08 \text{ g/kg}. \end{aligned}$$

d) The mixing ratio is defined as the ratio between the mass of the water vapour and the mass of dry air

$$\begin{aligned} r &= \frac{m_v}{m_z} = \frac{\rho_v}{\rho_z} = \frac{e}{p-e}\frac{R}{R_v} \\ &= 9.21 \text{ g/kg}. \end{aligned}$$

e) The dew point temperature T_d is defined as the temperature by which the current vapour pressure ($e = 14.59$ mbar) becomes saturated. Again, the Claussius-Clapeyron equation can be used, so that T is expressed and for e_s the current vapour pressure ($e = 14.59$ mbar) is used.

$$T_d = \left(\frac{1}{T_0} - \frac{R_v}{h_\mathrm{i}} \ln \frac{e}{e_{s0}}\right)^{-1}$$
$$= 285.5 \text{ K}.$$

7.2 Determine the relative humidity, if the temperature is 5 °C and the dew point temperature is −5 °C. The atmospheric pressure is 1000 mbar.

7.3 Calculate the dew point temperature under the following conditions: the temperature is 18 °C, the atmospheric pressure is 990 mbar and the relative humidity is 65%.

7.4 Determine the dew point temperature, if the relative humidity is 60%, the temperature −15 °C and the atmospheric pressure 1000 mbar?

7.5 How much lighter is one cubic metre of moist air with 90% relative humidity, from the air in which the relative humidity is only 10%? The temperature is in both cases 20 °C and the atmospheric pressure is 1000 mbar.

7.6 What is the mass of the water vapour in the classroom that is high 3 m, long 6 m and wide 5 m. The temperature is 22 °C, atmospheric pressure is 990 mbar and vapour pressure is 13 mbar?

7.7 What is the air density at the ground, if the atmospheric pressure is 1020 mbar and the temperature is 13 °C? What is its density, if the relative humidity of the air is 70%?

7.8 What is the specific humidity of air, if the vapour pressure is 5 mbar, the air density 1.1 kg/m^3 and the air temperature 10 °C?

7.9 What is the saturated specific humidity at 1000 mbar and the temperatures 30 °C and −15 °C?

7.10 How much does the relative humidity relatively change, if at the normal conditions:

a) the air temperature changes for 3 K and the vapour pressure does not change?

b) the vapour pressure of the water vapour changes for 1% (0.1 mbar) and the temperature stays the same?

7.11 We are cooling the moist air, which has a temperature of 30 °C and vapour pressure of 10 mbar. At which temperature will saturation occur? How much of water is condensed, per unit of the air volume, when the air is cooled to the final temperature of −16 °C?

7.12 The relative humidity of the air is 65% and the temperature is 15 °C. What will the new relative humidity be if the air would isobariclally heat up due to receiving 5000 J of the solar energy per every kilogram? The process takes place at the ground where the air has the density of 1 kg/m^3.

7.13 In the evening the measured air temperature is 20 °C and the relative humidity is 80%. Over the night, the air temperature near the ground will decrease by 7 K. Will dew form during the night on the ground?

7.14 The temperature and dew point temperature of the 20-m thick air layer at the ground are 20 °C and 10 °C. The ground is moist and the water evaporates into the air. What is the mass of the evaporated water in every quadratic metre of the ground, if you assume that the ground level air becomes saturated and that there is no mixing with its surroundings? The atmospheric pressure is 1000 mbar.

7.15 During the winter, we sometimes ventilate our apartment. Into the room in which the temperature is 20 °C and 50% relative humidity, we let in the outside air, which has the temperature −5 °C and 90% relative humidity. The cold air replaces half of the volume of the warm air in the room. The air is then mixed and the mixture is again heated to 20 °C while the atmospheric pressure remains constant at 1000 mbar. What is the final relative humidity?

7.16 How much of the water has to evaporate from the evaporator, if we want to achieve a 70% humidity in the room, which is 4 m × 3 m × 2.5 m large and has a constant temperature of 25 °C and the initial relative humidity of 45%. Assume a constant atmospheric pressure of 1000 mbar.

7.17 At sunset, the temperature is 15 °C and the relative humidity is 80%. On a clear night, the air is cooling by 1 K per hour. If the night lasts for ten hours, will there be dew and fog in the morning?

7.18 Two air masses are uniformly mixed so that in every kilogram of air, there is exactly half kilogram of air from one and half kilogram of air from the second air mass. Mixing takes place at the constant atmospheric pressure of 1000 mbar. The temperature of the warm air is 21 °C, and the temperature of the cool air is 5 °C. The warm air is saturated, and in the cool air the relative humidity is 80%. Determine the temperature after the mixing. Did condensation occur and if it did, how much water did condense?

7.19 The air has the temperature 10 °C and the atmospheric pressure is 1000 mbar. How much of the water is condensed from each cubic metre of the saturated air if the air cools by 1 K?

7.20 The temperature in a cloud increases from 0 °C to 5 °C. How much does the vapour pressure change, relative humidity at the end reaches 50%? The cloud is at the constant atmospheric pressure of 400 mbar.

7.21 Calculate the saturated vapour pressure at −12 °C over water and over ice?

7.22 What is the ratio of the relative humidity over the water and over the ice at a temperature of −5 °C?

7.23 How is the relative humidity changing, if the temperature at the ground is changing sinusoidally with an amplitude of 10 K, has the maximum at 14:00 and the minimum at 8:00 and the average temperature is 15 °C. The specific humidity and the atmospheric pressure are constant, at 3 g/kg and 1020 mbar, respectively. What will be the relative humidity at 10:00?

7.24 Calculate the relative humidity, if you the temperature of the dry bulb thermometer is 12 °C and the temperature of the wet bulb thermometer is 10 °C. Assume an atmospheric pressure of 960 mbar.

7.25 Calculate the dew point temperature, if you know the temperature of the dry bulb thermometer is 20 °C and the temperature of the wet bulb thermometer is 12 °C and the atmospheric pressure is 1020 mbar.

7.26 Calculate the temperature of the wet bulb thermometer at the atmospheric pressure of 1000 mbar, when the temperature is 18 °C and the relative humidity is 65%.

7.27 When the temperature is 10 °C, it starts raining with an intensity of 5 mm/h. At the beginning of the rain, a bridge has a temperature of −2 °C, that is why icing starts to form on it. The bridge is 20 cm thick. The specific heat capacity of the bridge is 800 J/kg K and the density 2500 kg/m^3. The drops have the same temperature as the air.

a) How long should the rain fall before the bridge heats up by 1 °C?

b) How long should the rain fall before there is no more ice on the bridge?

7.28 What is the mass of the precipitation, that is intercepted by the rain gauge over a time of three hours? Assume the horizontal wind speed is 10 m/s, the falling speed of the rain drops is 18 m/s and that in each cubic metre of the air, there is 1 g of water in liquid state. The surface area of an ombrometer is 4 dm^2. What is the mass of the precipitation, if instead of raindrops, snowflakes are falling at the speed of 5 m/s?

7.29 Calculate the virtual temperature of the air (T_v) at 30 °C, if the specific humidity is $20 \cdot 10^{-3}$. The virtual temperature is the temperature that dry air would have, if its density were the same as for moist air.

Chapter 8

Adiabatic changes

8.1 At which height will a cloud base form, if the air is rising from the ground upwards. At the ground the air temperature and dew point temperature are 15 °C and 11.6 °C?

Solution:
The rising air is cooling at 10 K/km ($\Gamma_a = 10$ K/km). Γ_a is by definition always positive, although the temperature of the rising air decreases with height. When using Γ_a, one therefore needs to be careful to prevent errors due to the incorrect usage of a positive or negative sign.

Besides the temperature, also the dew point temperature is decreasing in the rising air. The decrease is approximately equal to $\frac{1}{6}\Gamma_a$.

The cloud base is at a height, where the temperature of the rising air is equal to the dew point temperature of this air.

$$T - \Gamma_a \cdot z_B = T_d - \frac{1}{6}\Gamma_a \cdot z_B$$
$$z_B = 1.4 \text{ km}$$

8.2 Part of the air, which is close to the ground, is overheated by 10 K above the ambient temperature and is rising in the standard atmosphere. What is the hydrostatically unbalanced part of the specific buoyancy force 500 m above the ground?

Solution:
The air temperature at the ground in the standard atmosphere is $T_{00} = 288.15$ K and is decreasing with height at a rate of 6.5 K/km ($\frac{\partial T}{\partial z} = -0.0065$ K/m).

The temperatures of the rising and surrounding air at 500 m above the ground are:

$$T_{ok} = T_0 + \left(\frac{\partial T}{\partial z}\right)\Delta z = 284.9 \text{ K},$$
$$T = T_0 - \Gamma_a \Delta z = 293.15 \text{ K}.$$

The hydrostatically unbalanced part of the specific buoyancy force is:

$$\frac{dw}{dt} = g\frac{T - T_{ok}}{T_{ok}} = 0.28 \text{ m/s}^2.$$

8.3 The air, which has the temperature of 15 °C and the specific humidity 1.1 g/kg at the ground, is rising to the height of 6000 m because of the unbalanced buoyancy. What will the temperature of air at this height be and at which height will the air become saturated? The altitude of the ground is 0 m and the atmospheric pressure at the ground is 1000 mbar; the moist adiabatic lapse rate is 7 K/km.

8.4 From Radovljica, one can see that the base of the orographic cloud is at the height of 1700 metres on the slope of mountain Stol. A light southerly wind is blowing, and air from Radovljica is climbing along the slope of Stol. What is the relative humidity in Radovljica, if the air temperature there is 15 °C? The altitude of Radovljica is 450 m.

8.5 The wind is blowing at the speed of 8 m/s along an extensive slope, that has a tilt of 4 °C. Dry air travels from the bottom to the top of the hill in 26 minutes. How much does the temperature change on the top of the slope after the wind starts to blow, if the horizontal temperature gradient is negligible and the vertical component of the gradient is 5 K/1000 m? Evaluate the temperature change, if the air is saturated. Take into account the approximate value of the moist adiabatic lapse rate $\Gamma_s = 6$ K/km.

8.6 The wind is blowing from the valley up to the hill. In the valley, the temperature is 15 °C and the relative humidity is 80%. What is the relative humidity 500 m above the bottom of the valley?

8.7 A part of the air with temperature 17 °C and absolute humidity 7 g/m^3 is rising adiabatically. How much will the absolute humidity decrease, if the air rises by 1000 m without exchanging the heat or the humidity with the surroundings?

8.8 How much does relative humidity of the air chanege, if the air rises by 500 m at the constant specific humidity of 5 g/kg. The temperature at the ground is 20 °C and the atmospheric pressure is 1000 mbar. Calculate the height of the condensation level.

8.9 The air is raised adiabatically from 1000 mbar and 28 °C, to 850 mbar, where it becomes saturated. What is the relative humidity of air at the ground? At 200 metres above the level of condensation the temperature decreases by 1 K and the atmospheric pressure by 18 mbar. How many grams of the water are condensed per unit of the air mass if the air rises by an additional 200 m?

8.10 Calculate the height of the free convection of the unsaturated air, if the vertical temperature gradient is 6.5 K/km and the air near the ground warms up by 5 °C.

8.11 In the morning, the atmosphere above the airport is stable, with temperature decrease of 3 K per 1000 m. The temperature at the ground was 10 °C and the dew point temperature was 2 °C. During the day, the air near the ground becomes warmer and at 11:00 the temperature is 18 °C. To which height did the convection extend? Did cumulus clouds form? If they did, where was their base?

8.12 Dry air is rising from 1000 mbar to 700 mbar without mixing or exchanging the heat with its surroundings. At the beginning, it has a temperature of 10 °C. What is the initial density of the air? What are the final temperature and the density of the air?

8.13 The wind blows towards a 800-m high ridge and the air, which has a temperature of 15 °C and 70% relative humidity has to forcibly lift along the slope of the ridge. Does an orographic cloud form at the ridge? At which height will the cloud base be in case of 90% relative humidity?

8.14 In the morning, over the sea, the air temperature is 20 °C with a 3 K/km decrease to the height of 2 km. From there upward, a strong inversion exists, and the temperature is increasing by 2 K/km. To which height will the convection extend, if the air at the ground warms up to 27 °C during the day? Do the clouds appear, if the relative humidity at the ground would be 60% in the morning?

8.15 The air with the initial temperature 294 K and specific humidity 10 g/kg is rising at the hill from 1000 mbar to 700 mbar. What is the dew point temperature at 1000 mbar? At which atmospheric pressure is the lifted condensation level?

8.16 The air with a temperature of 18 °C and a relative humidity of 85% is flowing over the Alps. At the south side, it rises from the level of Jadransko morje to the top of the ridge (3000 m), at the north side it sinks to Bavaria (700 m) without precipitation fall out. What are the temperature and the relative humidity north from the Alps? Assume a moist adiabatic lapse rate of $\Gamma_s \sim 7$ K/km.

8.17 Over the mountains blows the foehn wind, which has the temperature of 38 °C and the mixing ratio of 4 g/kg at 1000 mbar. Can this be the same air as on the windward side of the mountain, where the temperature is 21.5 °C and the mixing ratio 10 g/kg at 1000 mbar?

8.18 The air at 20 °C and mixing ratio 8 g/kg is raised from 1000 mbar along the hill to 700 mbar. What was the dew point temperature before the rise? What is the temperature on the other side of the hill at 900 mbar, if 80% of the mass of condensed water falls from the cloud and the precipitation water does not evaporate into the air?

8.19 The air flows over the Alps (altitude 3000 m). On the southern side at the bottom, the air has the temperature of 18 °C and the relative humidity of 85%, on the northern side at the bottom the southerly foehn is blowing with the temperature of 25 °C and the relative humidity of 36%. How much precipitation falls when crossing the Alps?

8.20 In the lower troposphere, during peaceful dry weather, in the night we measure at the ground the temperature of 16 °C. The temperature is decreasing with height by 7 K on km to the height of 3 km and by 5 K on km to the height of 5 km. During the day, the air at the ground warms up to 28 °C. To which height does the air mix due to convection?

8.21 What is the frequency of vertical oscillation of vertically displaced unsaturated air in a stable atmosphere ($\partial T/\partial z = -5$ K/km, the temperature is 15 °C)? What happens, if $\partial T/\partial z = -11$ K/km?

8.22 The saturated air with the temperature of 20 °C and the atmospheric pressure of 1000 mbar blows at the speed of 10 m/s towards a slope with the tilt of 20°. What is the maximum possible intensity of the precipitation that falls from the rising air, if the cloud top is at the pressure 500 mbar? Assume moist adiabatic lapse rate of $\Gamma_s = \beta(p)\Gamma_a$:

atmospheric pressure (mbar)	β
1000	0.38
850	0.45
700	0.50
500	0.62

Chapter 9

Emagrams

9.1 The meteorological balloon measured the following temperatures at different heights:

atmospheric pressure	temperature	dew point temperature
1000 mbar	20 °C	10 °C
950 mbar	20 °C	10 °C
600 mbar	−14 °C	−40 °C
300 mbar	−45 °C	−60 °C
above 300 mbar	−45 °C	no data

a) Draw the vertical temperature profile of the Skew-T Log-P emagram (between the measurements you can draw the straight lines).

b) Draw the vertical profile of the dew point temperature on the emagram.

c) What are the temperature and the dew point temperature at the height, where the atmospheric pressure equals 450 mbar?

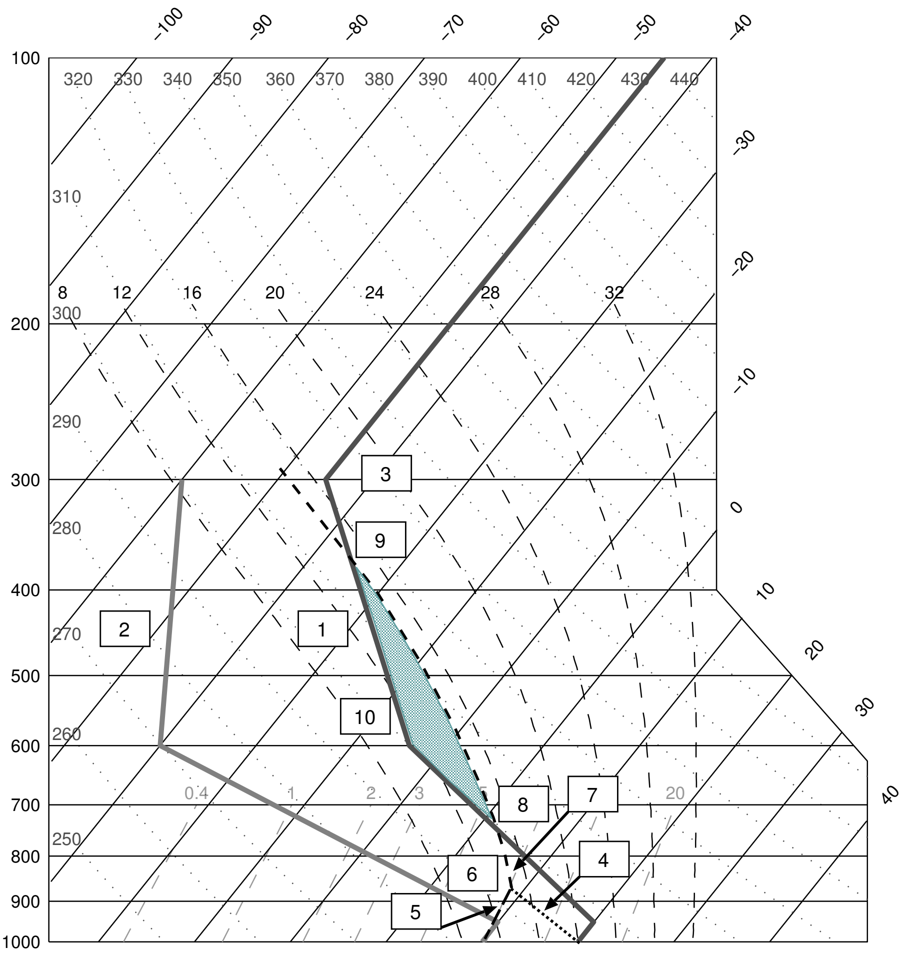
d) Determine the lower limit of the tropopause.

e) At which height will cloudiness appear, if the air is forced up along the slope? Draw the vertical profile of the temperature during the rising on the emagram.

f) To which height would the air have to be additionally raised so that the free convection will occur? Where will the cloud top be in this case? (Mark on the emagram.)

g) Mark the CAPE (Convective Available Potential Energy).

Solutio



a),b) On a Skew-T Log-P emagram the atmospheric pressure is on the ordinate axis (marked on the left side and goes from 1000 mbar to 100 mbar). The abscissa axis shows the temperature and is rotated by 45° in the clockwise direction (solid black lines inclined to the right). The temperature values are labelled at the right and top sides and go from 40 °C to −100 °C. The vertical profiles of the temperatures with the height (atmospheric pressure) that are given in the example are shown with thick lines. The right line represents the temperature and left the dew point temperature.

c) You have to read the temperature from the graph (points 1 and 2). The temperatures are approximately −27 °C (air) and −48 °C (dew point).

d) The tropopause is the isothermal layer, which is located approximately 10 km above the troposphere. For accurately determining its lower limit, you have to determine the height where the temperature becomes constant from the graph. In our case, that is at point 3 (300 mbar). At this point, the vertical profile of the air temperature becomes parallel to the inclined temperature axis.

e) If an air parcel is rising, its temperature and the dew point temperature are changing. The temperature of the rising air with the height decreases parallel with the dotted lines that are inclined to the left. When the air is rising from the ground, the temperature parallelly follows that lines (marked with 4). At the same time, the dew point temperature is decreasing. It is decreasing in parallel with the shorter grey dashed lines inclined to the right (marked with 5). When the air temperature and the dew point temperature become equal, the air becomes saturated (the relative humidity becomes 100%). This is labelled with 6. At that height, the cloud droplets will start to form and cloudiness appears. This height is also known as the rising condensation level.

f) From the rising condensation level, the air is rising parallel to the black dashed curve that is inclined to the left (labelled with 7). The dew point temperature is equal to the air temperature (the relative humidity is 100%). The rising air is still cooling, but somewhat more slowly than before the rising condensation level. If at some point, the temperature of the rising air becomes higher than the temperature of the surrounding air at current height, the unbalanced buoyancy force starts to point upwards and the free convection occurs. In our case this happens at the atmospheric pressure around 750 mbar (labelled with 8). From here on, the air will, due to the buoyancy, raise itself (it does not need to forcibly raise itself along the hill) until it will become colder from the surroundings. It will become colder than the surroundings at the atmospheric pressure around 360 mbar (labelled with 9). In this case the cloud will extend from the rising condensation level (point 6) to the upper level of the free convection (point 9).

g) Convective Available Potential Energy (CAPE) is defined as the surface area between the curves of the temperature of the rising and the surrounding air, where the first is warmer than the second (greyness labelled with 10). It is difficult to qualitatively evaluate the CAPE from the emagram, but it is generally true that the larger the CAPE surface area is, the more intense the convection will be (if the convection does occur – do not forget that the air first has to be forcibly raised at the slope of the hill to the height of 750 mbar. If there is no forced rising, the convection will not occur).

9.2 The meteorological balloon measured the following temperatures at different heights:

atmospheric pressure	temperature	dew point temperature
1000 mbar	34 °C	23 °C
950 mbar	30 °C	22 °C
900 mbar	25 °C	20 °C
850 mbar	22 °C	17 °C
800 mbar	21 °C	13 °C
750 mbar	18 °C	5 °C
700 mbar	12 °C	−1 °C
650 mbar	6 °C	−10 °C
600 mbar	2 °C	−12 °C
550 mbar	−3 °C	−21 °C
500 mbar	−7 °C	−25 °C
450 mbar	−12 °C	−30 °C
400 mbar	−17 °C	−36 °C
350 mbar	−25 °C	−39 °C
300 mbar	−32 °C	−42 °C
250 mbar	−42 °C	−51 °C
200 mbar	−52 °C	−60 °C
150 mbar	−51 °C	no data
100 mbar	−46 °C	no data

a) Draw the vertical temperatures profile on the blank Skew-T Log-P emagram.

b) What is the relative humidity of the surrounding air at the ground and at 700 mbar?

c) Determine the lower limit of the tropopause.

d) Did any cloud layers exist at the time of the measurement?

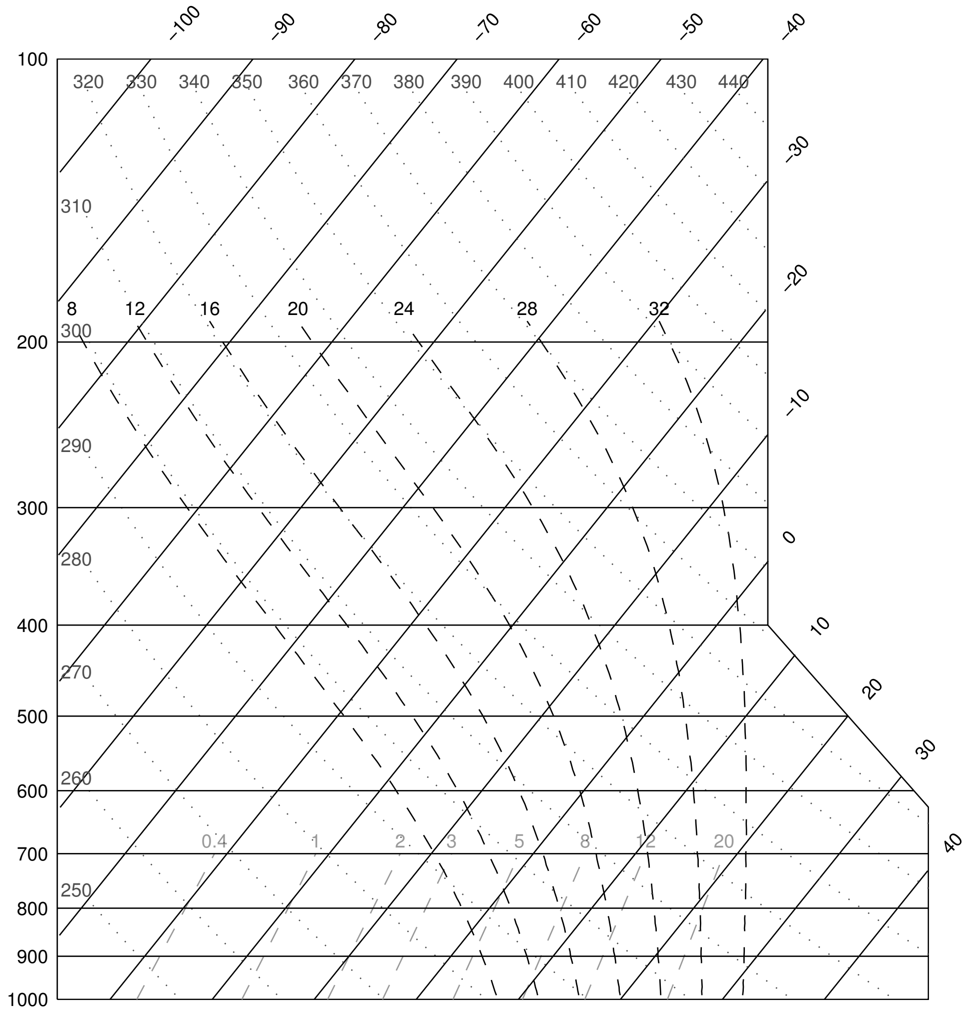
e) Determine the lifted condensation level.

f) Determine the height of the cloud base.

g) Determine the level of free convection?

h) In the case of free convection, to what height will the cloud reach?

i) How much would be the air at the ground have to warm up, so that free convection with condensation would occur?



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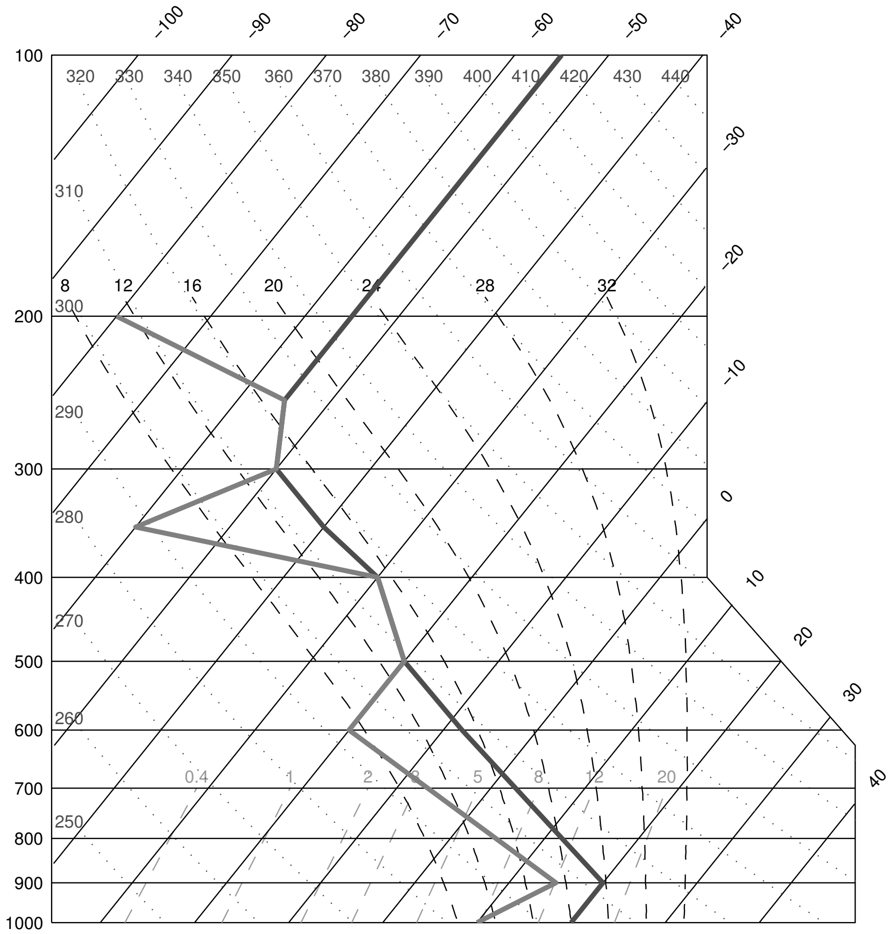
9.3 Using the provided Skew-T Log-P emagram determine the following:



a) For how many degrees does the temperature decrease in the layer between 800 mbar and 500 mbar?

b) Mark on emagram where the cloud layers were located at the time of the measurement.

c) On the emagram mark the lower limit of the tropopause.

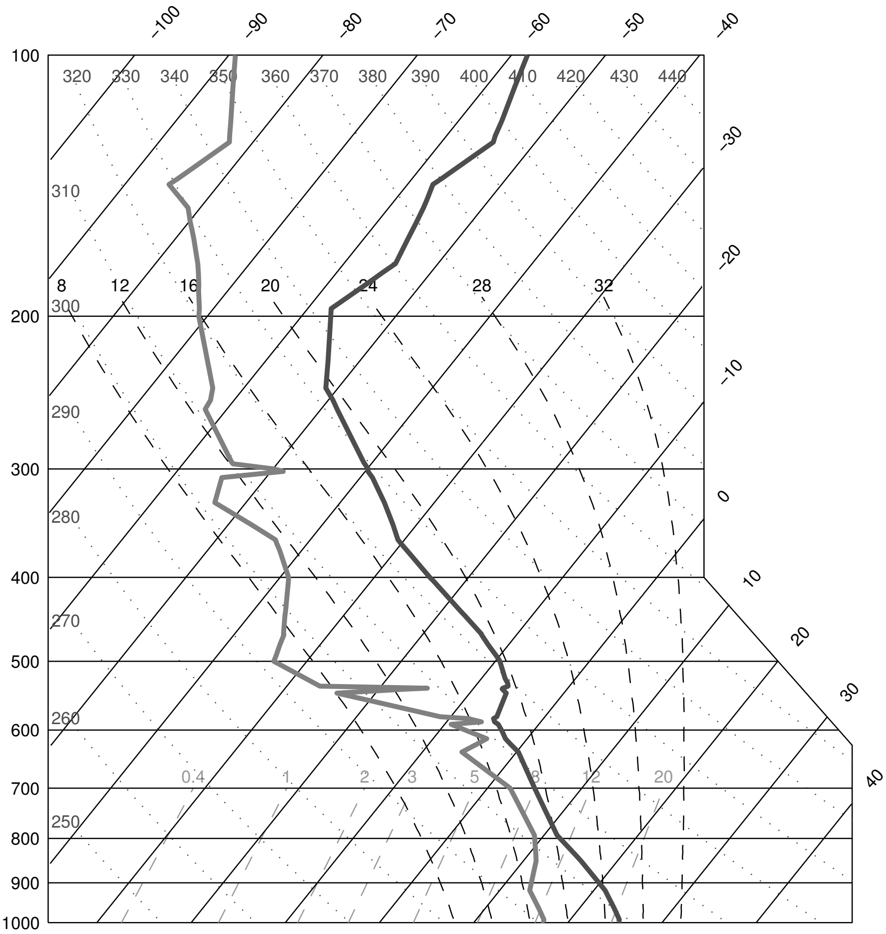
d) On the emagram mark and evaluate the lifted condensation level.

e) On the emagram mark to which height the air would additionally need to be raised so that the free convection would occur?

f) For how many degrees should the air at the ground warm up, so that the free convection with the condensation will occur?

g) On the emagram mark, to which height would the cloud extend in this case.

9.4 Using the provided Skew-T Log-P emagram determine the following:



a) What is the temperature at the height of 700 mbar?

b) Did any cloud layers exist at the time of the measurement?

c) On the emagram mark and determine the height of the lifted condensation level.

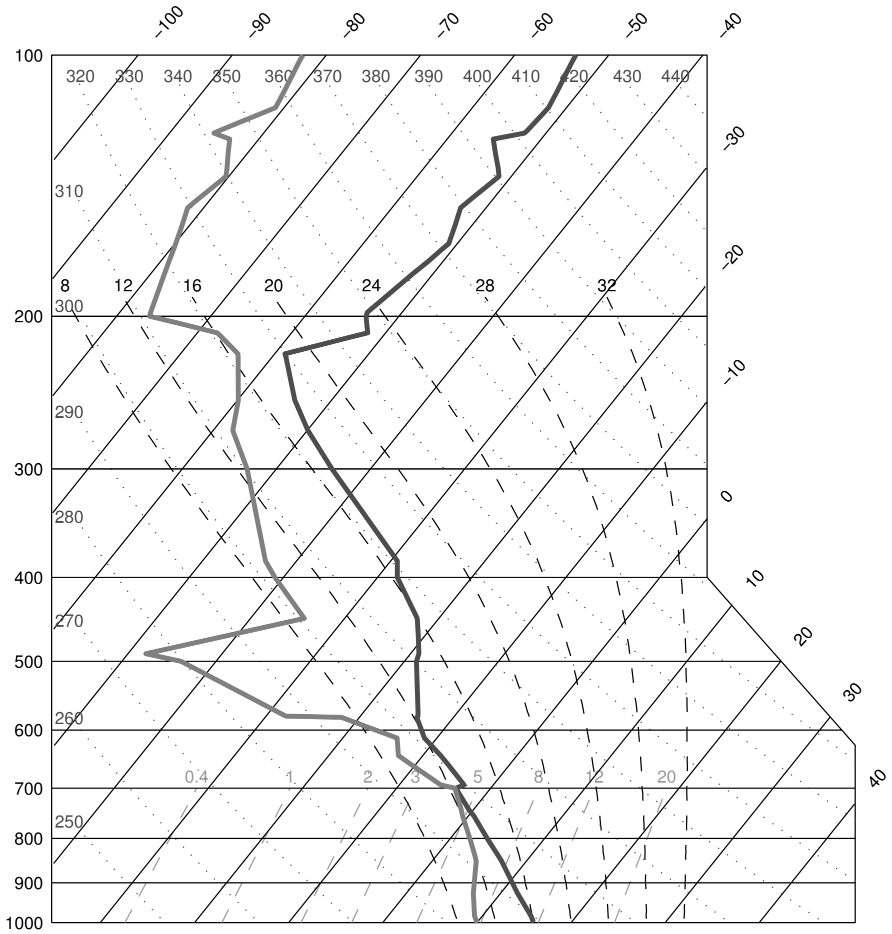
d) The air is forcibly rising at the slope of the hill, that has the top at 850 mbar. Will the free convection occur?

e) To which height will the cloud extend in this case?

f) By how many degrees should the air at the ground warm up, so that the free convection with the condensation will occur?

g) To which height will the cloud extend in this case?

9.5 Using the provided Skew-T Log-P emagram, determine the following:



a) What is the temperature at the height of 450 mbar?

b) Did any cloud layers exist at the time of the measurement?

c) On the emagram, mark and determine the height of the lifted condensation level.

d) On the emagram, mark to which height the air would additionally need to be raised so that the free convection would occur?

e) At least for how many degrees should the air at the ground warm up, so that the free convection that will take place to the height of 200 mbar will occur?

f) To which height will the cloud extend in this case?

g) Estimate the CAPE when the temperature of air near the ground increases to 20 °C. The integral can be approximated by a sum over a few layers.

Chapter 10

Radiation

10.1 Calculate the ratio between the incoming power of solar radiation for sunny and shady parts of a roof with a tilt of 30°. The ridge of the roof is in the east-west direction, the sun is positioned in the south at the elevation of 45° above the horizon. The flux density of the incoming direct solar radiation is 800 W/m^2 and the flux density of the diffuse radiation is 44 W/m^2.

Solution:
The sunny and the shady sides are receiving power:

$$P_{\text{sunny}} = j_{\text{dir}} \cdot \cos\left(\frac{\pi}{2} - (\alpha + \gamma)\right) \cdot S + j_{\text{dif}} \cdot S$$
$$P_{\text{shady}} = j_{\text{dir}} \cdot \cos\left(\frac{\pi}{2} - (\alpha - \gamma)\right) \cdot S + j_{\text{dif}} \cdot S$$

The ratio of the energy fluxes is: $P_{\text{sunny}}/P_{\text{shady}} = 3.3$.

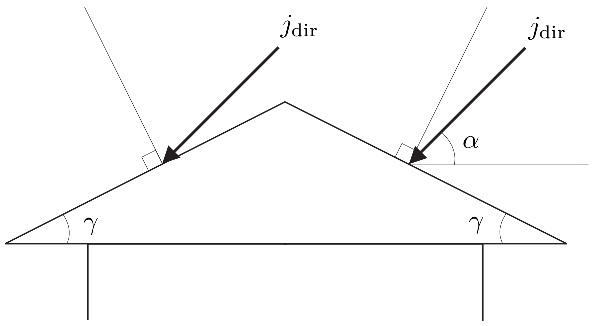


Figure 10.1: Illustration of first probem

10.2 The temperature of all walls in a prison cell is 10 °C. In the cell is a naked prisoner and the temperature of the human skin is 30 °C. What are the net radiative energy loses of the prisoner, if you assume, that the surface area of the human body is 1 m^2 and the walls and the human body radiates as a black body?

10.3 Fog has risen from the ground to a height of a few tens of metres. The emissivity of the fog is 0.8 and temperature 282 K. What is the power of the incoming radiation that is received by a flat surface on the ground? The area of the surface is 1 m^2. No IR radiation is absorbed between the fog and the ground.

10.4 The estimate of the average evaporation of the water for the whole Earth is 2.7 mm/day. This corresponds to the average precipitation. What portion of the incoming average solar radiation received by the Earth is needed for the evaporation of the water? Assume the average albedo of the Earth $a = 0.35$.

10.5 How many days would the sun have to irradiate the horizontal plane on the Earth surface, that 1200 mm of water would evaporate from it? Assume the average length of the day is 12 h and the height of the Sun at noon is 60° above the horizon. Assume a linear function for the sun's elevation during the day.

10.6 Assume the Earth's surface and the atmosphere radiate according to Planck's law. The planet as a whole on average emits as much energy as it receives from the Sun. In the atmospheric window, between 8 and 11 μm, the radiation from the ground proceeds freely into space while in other spectral regions the atmosphere absorbs all radiation. The atmospheric window contains about 30% of the energy emitted by a black body with the temperature of the ground. What is the average temperature of the upper layers of the atmosphere in the case of radiative balance? The average temperature of the ground is 15 °C. The average albedo of the Earth is $a = 0.35$, the emissivity of the ground $\varepsilon_{\text{ground}} = 1$ and the emissivity of the atmosphere $\varepsilon_{\text{atm}} = 0.7$.

10.7 A nuclear power station uses a nearby lake for cooling and is dumping waste heat with the power of 500 MW. The lake has a surface area of 100 km^2 and the average depth of 10 m. Assume that the waste heat is evenly distributed over the entire lake.

a) How fast would the lake warm up, if it would not emit the energy to the surrounding area?

b) If the lake has a natural temperature of 10 °C, how will its temperature change, if the surplus of the heat will be emitted only via the emited radiation?

c) How much would the evaporation have to be increased to account for all the waste heat?

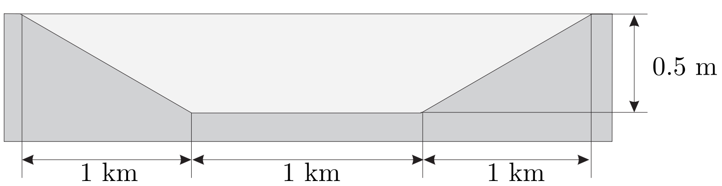
10.8 A meadow with the surface area of 1.2 ha, absorbs 18000 MJ of the energy from solar radiation and evaporates 1000 kg of water in one hour. If the emissivity of the meadow ground is 0.95, what is the temperature of the ground on the meadow?

10.9 In Greenland, a circular lake with the radius of 1 km and the depth of 0.5 m exists. On a polar day, the Sun is at elevation 25° above the horizon all the time and the flux density of direct solar radiation is 800 W/m^2. 80% of the incoming solar radiation is absorbed in the water. All the absorbed energy is used only for the evaporation of the water.

a) In what time would the lake dry out, if the situation stays unchanged and there is no precipitation?

b) How many days would pass before the lake would dry out, if it has a constant river inflow with the capacity of 500 m^3/h.

10.10 In Greenland, a circular lake with the cross-sectional profile as shown in the figure below exists. On a polar day, the Sun is at elevation 25° above the horizon all the time, and the flux density of direct solar radiation is 800 W/m^2; 80% of the incoming solar radiation is absorbed in the water. All the absorbed energy is used only for the evaporation of the water.

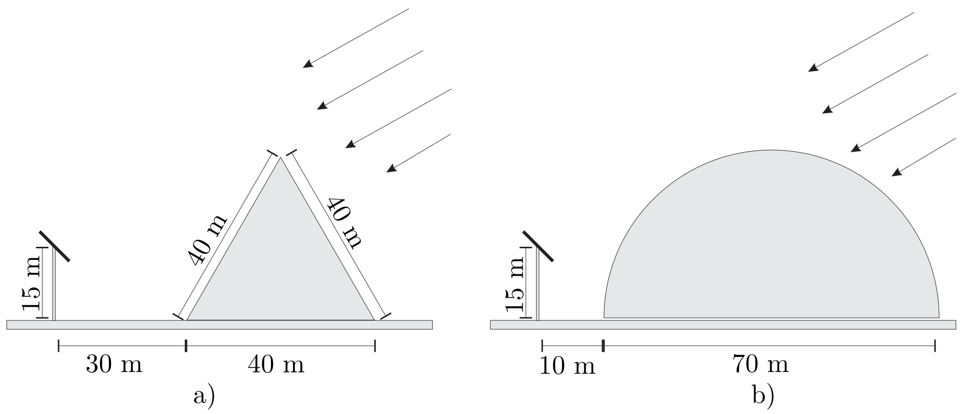


a) In what time would the lake dry out?

b) What would be the equilibrium depth of the lake, if the lake would have constant river inflow with the capacity of 500 m^3/h?

c) In what time would the lake dry out, if the lake would have constant river inflow with the capacity of 200 m^3/h?

10.11 On a 15-m high pillar, there is a solar collector with the surface area of 1 m^2. The collector is tilted at 45° with respect to the horizon. Between the collector and the Sun is a triangular or a half-circular hill (see figure). What is the power of the incoming solar radiation that falls on the collector in the morning when the Sun shines on it? Assume that the flux density of direct solar radiation is 800 W/m^2.



10.12 During the day, the temperature of the soil surface is changing sinusoidally with the amplitude of 10 K, with maximum at 14:00 and minimum at 8:00 hour. The average temperature of the soil surface is 15 °C. What is the amplitude of the temperature at the depth of 15 cm in the soil? Assume thermal conductivity of soil $\lambda = 100$ W/m^2, soil density 2000 kg/m^3 and specific heat capacity of soil of 2000 J/kg K?

10.13 The albedo of Earth with the atmosphere is 0.3 and the atmosphere absorptivity is 0.1 for the shortwave and 0.7 for the longwave radiation. What are the equilibrium temperatures of the ground and the atmosphere, if the solar constant j_0 is known?

10.14 Assume that some area on the Earth is isolated from its surroundings, so that the energy fluxes between this area and the surroundings can be neglected. This area is irradiated by the Sun; near the ground the flux density of solar radiation is 600 W/m^2. The albedo of the soil is 0.36. The atmosphere consists of the three layers, which are not mixing with each other. We neglect the transfer of energy with the conduction between the nearby layers. The transmissivity of the three layers is 0.70 and the reflectivity 0. What will the ground temperature be, if the layers and the ground are in the radiative balance? Assume, that the layers and the ground affect only the neighbour.

10.15 How much energy of solar radiation is received by a black horizontally oriented plate of the radiometer, with surface 6 cm^2 at the time between 11:30 and 12:30 on the local solar time, when the Sun is at noon 60° above the horizon? What is its equilibrium temperature at noon? What is the total daily received energy, if the Sun rises at 5:45 and the elevation angle of the Sun is changing sinusoidally?

10.16 A thermometer is placed in direct sunlight and shows the temperature of 67 °C. The true air temperature is 25 °C, while the emissivity of air is 0.3 and the thermometer radiates as a black body. What is the reflectivity of the thermometer bulb, if the incoming flux density of solar radiation is 400 W/m^2 and the surface, irradiated by the Sun, is 1 cm^2.

Chapter 11

Fronts

11.1 What is the tilt of the cold front, if the difference in the temperature between the air masses is 3 K, a south-westerly wind is blowing before the front at the speed of 13 m/s and after the front a north-westerly wind is blowing at the speed of 10 m/s? The front is oriented in the north-south direction. The temperature of the warm air is 16.5 °C. The front is located at 45° N.

Solution:
We use the equation for the tilt of the front:

$$\tan\alpha = \frac{f\overline{T}}{g}\left(\frac{v_{\mathrm{T}} - v_{\mathrm{H}}}{T_{\mathrm{H}} - T_{\mathrm{T}}}\right),$$

where v_{T} and v_{H} are the tangential components of the wind, which blows along the front. $\tan\alpha$ is defined as positive in cold front and negative in the warm front. In the example, we firstly need to determine the tangential components of the wind, that are $v_{\mathrm{T}} = -13\ \mathrm{m/s}\cdot\sin 45°$ and $v_{\mathrm{H}} = 10\ \mathrm{m/s}\cdot\sin 45°$.

$$\tan\alpha = 0.0164.$$

11.2 How fast will the cold front move from Koper towards central Slovenia, if in the warm air the wind blows from the azimuthal direction 210° at the speed of 25 m/s and in the cold air the atmospheric pressure increases towards the southwest by 1.3 mbar/100 km? The air density is 1 kg/m^3.

11.3 The front is located on the northern hemisphere in the east-west direction and is vertically tilted by 1°. In the warm air the temperature is 6 °C and in the cold air −2 °C. In the cold air, the wind is blowing from the northeast at the speed of 40 m/s. Determine the wind blowing in the warm air.

11.4 What is the orientation of the front and how fast is it moving if in cold air northerly wind at the speed of 10 m/s is blowing while in warm air north-westerly wind at the speed of 7.1 m/s is blowing?

11.5 At the cold front with a tilt of 1/200 and orientation in the SW-NE direction, the cold air is advancing at the speed of 50 km/h. The cold air is 5 K colder than the warm air. In the warm air, where the temperature is 15 °C, a westerly wind blows. What are the speed and the wind direction in the cold air?

11.6 A cold front is approaching Slovenia from the west. The front is orientated in the north-south direction. Before the front blows the westerly wind with 10 m/s and after it the north-northeasterly wind. The temperature in the warmer air is 10 °C and in the colder 5 °C.

a) Calculate the speed of the wind in the cold air and the tilt of the front.

b) In Koper, the north-northwesterly wind started to blow at 3:00 in the morning. When can we expect the turning of the wind in Ljubljana? Koper lies at azimuth 230° with respect to Ljubljana and the distance between the two is 95 km.

11.7 An isothermal lake of cold air (T_1 = 5 °C) lies in the basin where the weather is calm. Above the inversion that separates the lower air mass from the upper air mass, the air is warmer (T_2 = 15 °C) and westerly geostrophic wind at the speed of 15 m/s in blowing. At the side of the basin, where the inversion level is the lowest, the inversion is 100 m high. At which height is the inversion on the opposite side of the basin, which is 40 km away. In which direction is the opposite side?

11.8 What is the slope of the upper limit of the calm lake of the cold air, in which the temperature is −8 °C, if above it wind speed is 10 m/s and temperature 5 °C?

11.9 Determine the tilt of a stationary front, which separates the two air masses at the latitude of 45°. Characteristics of the air masses: the atmospheric pressure at the ground is 1000 mbar, the temperature in the cold air is 263 K, and in the warm air 273 K. The horizontal gradient of the atmospheric pressure, perpendicular to the front in the cold air, is 1 hpa/100 km. In the warm air, the geostrophic wind blows at the speed of 15 m/s. How much would the tilt change if the warm air would stop moving?

11.10 How far ahead of the warm front at the ground is the warm air at the height of 8 km (usually we see it via cirrus clouds)? Assume the incoming air is 10 K warmer than the cold air, that its temperature is 5 °C and that at the front the wind turns by 45° and strengthens by 5 m/s? How far away on the wester horizon can this cloudiness be seen, if we stand on the hill 1000 metres high? Before the front, the wind is blowing at the speed of 10 m/s perpendicular to the direction of the front movement.

Chapter 12

TAF and METAR weather reports

12.1 Explain the meaning of the following meteorological weather report:
METAR EHAM 1050Z 24015KT 9000 RA SCT025 BKN040 10/09 Q1010 NOSIG=

Solution:
The measurement, that was performed at 10:50 UTC at EHAM (Amsterdam) airport, reports that the wind was measured from the direction of 240° at a speed of 15 knots. The visibility was 9 km, it was raining; at the height of 2500 feet the cloudiness was 3/8 to 4/8 and at the height of 4000 feet 5/8 to 7/8. The temperature was 10 °C and the dew point temperature 9 °C. The atmospheric pressure, reduced to the sea level, was 1010 mbar. In the next two hours, we do not expect significant changes in the weather.

12.2 Explain the meaning of the following meteorological weather report:
METAR EGLL 0920Z 26005KT CAVOK 15/14 Q1013 NOSIG=

12.3 Explain the meaning of the following meteorological weather report:
METAR EDDL 1550Z 26005KT 0550 R23L/0450 FZFG OVC002 M02/M02 Q0994 BECMG OVC005=

12.4 Explain the meaning of the following meteorological weather report:
METAR EIDW 0900Z 24035G55KT 210V270 1700 +SHRA BKN007 OVC015CB 08/07 TEMPO 3500=

12.5 Explain the meaning of the following meteorological weather report:
TAF CYYC 192038Z 192118 17008KT P6SM SCT020 OVC080 TEMPO 2203 P6SM -SHRA
BECMG 2223 24007KT
FM0300Z 32010KT P6SM SCT007 BKN060
FM0600Z 33015KT P6SM SCT010 BKN040 TEMPO 0612 5SM -RASN BR OVC010
FM1200Z 34015G25KT P6SM SCT010 OVC030 TEMPO 1218 2SM -SHSN OVC010
RMK NXT FCST BY 00Z=

Solution:
The forecast was issued on the 19th day of the month at 20:38 UTC for the CYYC (Calgary International Airport) airport. The forecast is valid for a period from the 19th at 21:00 UTC to the next day at 18:00 UTC. In the first period of the forecast (from 21:00 UTC to 03:00 UTC), the wind will blow from the direction of 170° at the speed of 8 knots. The visibility will be at least 6 statute miles (unit for the length that is in use in Canada. In Europe, metres are in use). At the height of 2000 feet, the cloudiness will be from 3/8 to 4/8. At the height of 8000 feet, the cloudiness will be 8/8. For the time between 22:00 UTC and 03:00 UTC light rain showers with unchanged visibility are predicted.

For the time between 22:00 UTC and 23:00 UTC the change of the wind to 240° and the speed of 7 knots is predicted.

Starting from 03:00 UTC the wind will blow from 320° at the speed of 10 knots. The visibility will not fall below 6 statute miles. Cloudiness from 3/8 to 4/8 will lower to 700 feet. At the height of 6000 feet, cloudiness ranging from 5/8 to 7/8 will appear.

Starting from 06:00 UTC the wind will blow from 330° at the speed of 15 knots. The visibility will not fall below 6 statute miles. Cloudiness from 3/8 to 4/8 will rise to 1000 feet. Cloudiness from 5/8 to 7/8 will lower to 4000 feet. From 6:00 UTC to 12:00 UTC, visibility will temporarily be reduced to 5 statute miles, and light rain with snow, fog and the cloudiness 8/8 at the height of 1000 feet are predicted.

Starting from 12:00 UTC, the wind will blow from 340° at the speed of 15 knots and the wind gusts at the speed of 25 knots. The visibility will not fall below 6 statute miles. Cloudiness from 3/8 to 4/8 will stay at the height of 1000 feet. Cloudiness 8/8 will be at the height of 3000 feet. From 12:00 UTC to 18:00 UTC, visibility will temporally be reduced to 2 statute miles and light showers with snow will appear. Cloudiness of 8/8 will be at the height 1000 feet.

The next forecast will be issued at 00:00 UTC.

12.6 Explain the meaning of the following meteorological weather report:
TAF LJLJ 040600Z 040716 VRB03KT 1500 -SN FEW001 BKN005 OVC020 TEMPO 0716 0800 SN=

12.7 Explain the meaning of the following meteorological weather report:
TAF LJMB 110530Z 040716 VRB01KT 3000 -RASN OVC033 BECMG 0709 1500 -SN FEW005 BKN020 OVC033 TEMPO 0716 0800 SN BECMG 1215 02010KT=

Chapter 13

Solutions

Units

1.1: 100000 Pa, 600 mbar, 0.5 bar.

1.2: 285.5 K, 17 °C, 259 K.

1.3: 259200 s, 2.3 years, 0.0009 km^2, 0.003 m^3, 108 km/h, 27.7 m/s, 10^8 W, $1.4 \cdot 10^9$ W/km^2, 18 °C/hour, 0.0015 Pa/m, 27.8 kWh, 18 MJ.

Structure and atmospheric layers

2.2: 355 kg.

2.3:

$$M = \frac{\Sigma m_i}{\Sigma n_i} = \frac{\Sigma m_i}{\Sigma \frac{m_i}{M_i}} = \frac{\Sigma \frac{V}{R^* T} p_i M_i}{\Sigma \frac{V}{R^* T} p_i}$$
$$= \frac{\Sigma p_i M_i}{\Sigma p_i} = 28.8 \text{ g/mol.}$$

2.4: a) 1.2 kg/m^3, b) 1.4 kg/m^3.

2.5: Air density increases from 1.02 kg/m^3 to 1.10 kg/m^3.

2.6: The partial pressure of dry air reduced for the partial pressure of water vapour is 994.9 mbar. The mass fraction of argon is 1.28%, so the partial pressure of argon is 12.7 mbar. From the gas equation for the ideal gases, it follows that the total mass of argon is:

$$m = \frac{p_{\mathrm{Ar}} V}{R_{\mathrm{Ar}} T} = 2.1 \text{ kg.}$$

2.7: The average atmospheric pressure at sea level is 1013 mbar. The radius of the Earth is 6370 km.

$$p = \frac{F}{S} = \frac{mg}{4\pi R_z^2},$$
$$m = 5.3 \cdot 10^{18} \text{ kg.}$$

Hydrostatic

3.3:

z	1 km	2 km	3 km	4 km	5 km	6 km	7 km	8 km	9 km	10 km
$p(z)/p_0$	0.88	0.78	0.68	0.59	0.51	0.44	0.38	0.32	0.27	0.22

3.4: The problem is solved progressively from the lower layer upwards.

height (m)	atmospheric pressure (mbar)	temperature (K)
0	1013	288
100	1001.1	290
1000	899.2	283
3000	706.2	283
12000	208.0	224.5
19964	10	224.5

3.5: a) 1028.8 mbar, b) 1026.2 mbar.

3.6: a) 1098.0 mbar, b) 663.2 mbar, 0.82 kg/m^3 c) 352.4 mbar, d) 1454 m.

3.7: a) 2853 m, b) 2816 m.

3.8: 3187 m.

3.9: 15781 m.

3.10: Using the parameters of standard atmosphere the atmospheric pressure at the height of the plane can be calculated (306.8 mbar). Then the actual data can be used to calculate the real height of the plane (8119 m).

3.11: We can calculate the pressure on Kredarica in two ways: first with $\left(\frac{\partial T}{\partial z}\right)$ for the standard atmosphere, where for the height of Kredarica we use the value shown by the altimeter, and second with the use of true temperature and height data from the temperature profile. The pressure increases for 1.5 mbar.

3.12: The layer is located between 866 m and 1818 m.

3.13: 2863 m.

3.14:

$$\begin{aligned}
dQ &= mc_p dT \\
&= \rho S h c_p dT, \\
\Delta z = z_1 - z_0 &= \frac{R}{g}\overline{T}\int \mathrm{d}\ln p, \\
d\Delta z &= \frac{R}{g}\ln\frac{p_0}{p_1}\,d\overline{T} \\
&= \frac{R}{g}\ln\frac{p_0}{p_1}\frac{1}{\rho h c_p}\frac{dQ}{S} \\
&= \frac{R}{\Delta p}\ln\frac{p_0}{p_1}\frac{1}{c_p}\frac{dQ}{S} = 2.9\text{ m.}
\end{aligned}$$

3.15:

$$d\Delta z = \frac{R}{\Delta p}\ln\frac{p_0}{p_1}\frac{1}{c_p}\frac{dQ}{S} = 63\text{ m.}$$

3.16: 280 m.

3.17:

$$p = \rho_{\mathrm{Hg}} g h = 974.3 \text{ mbar}.$$

3.18: By calculating the atmospheric pressure at the sea level it is assumed that the atmosphere is isothermal with the temperature measured at the station:

$$\frac{\Delta p}{p_0} = \mathrm{e}^{\frac{g\Delta z}{RT}} - \mathrm{e}^{\frac{g\Delta z}{R(T+1\text{ K})}} = 1.2 \cdot 10^{-4}.$$

3.19: 743.4 mbar.

3.20: In the case of homogeneous atmosphere, the density is constant and $dp = \rho R dT$. We can use the hydrostatic equation and use it to express the change of the temperature with the height:

$$\frac{\partial p}{\partial z} = -\rho g,$$
$$\frac{\partial T}{\partial z} = -\frac{g}{R} = -0.03 \text{ K/m}.$$

3.21: When there is no inversion in the layer above the ground, the temperature at the ground is -2.8 °C. The ratio of the calculated values of the atmospheric pressure is

$$\frac{p_0}{p_0'} = \mathrm{e}^{\frac{gh}{R}\left(\frac{1}{T} - \frac{1}{T}\right)} = 1.0003.$$

3.22: First, we calculate the height of the 500-mbar layer, by assuming the standard atmosphere: $p_0 = 1013$ mbar, $T_0 = 288$ K, $\left(\frac{\partial T}{\partial z}\right) = -6.5$ K/km.

$$z = \frac{T_0}{\left(\frac{\partial T}{\partial z}\right)} \left[\left(\frac{p_1}{p_0}\right)^{-\frac{R\left(\frac{\partial T}{\partial z}\right)}{g}} - 1 \right] = 5567 \text{ m}.$$

When calculating the geopotential, we assume that g is not changing with height

$$\Phi = \int_0^H g \,\mathrm{d}z = 54612 \text{ m}^2/\text{s}^2.$$

Basic laws

4.2:

$$\omega^2 R = 0.1 f u,$$
$$R = 20.7 \text{ km}.$$

4.3: It points towards the southeast. The force is $3.1 \cdot 10^{-3}$ m/s^2.

4.4: We have to take into account the decrease of gravitational acceleration with the height and the radial component of the centrifugal force. The mountaineer feels lighter by 0.37%.

4.5: The difference is due to the radial component of the Coriolis acceleration, which points in the opposite direction as the gravitational acceleration. In Slovenia ($\varphi = 45°$), this difference is 0.0123 m/s^2 and on the equator 0.017 m/s^2. For a human with the mass of 75 kg, this means 0.92 kg or 1.3 kg less.

4.6:

$$\frac{|-\rho g|}{|\frac{\partial p}{\partial n}|} = 7240.$$

4.7: The balloon will not be moving in the vertical direction, if the sum of the forces that acts upon it in the vertical direction is equal to zero. The gravitational force and the buoyancy force are balanced:

$$0 = -m_b g - m_k g + V_b \rho_{ok} g = \left(\frac{\rho_{ok}}{\rho_{zr}} - 1\right) - \frac{m_k}{\rho_{zr} V_b}.$$
$$V_b = \frac{R_s m_k}{p(T_{ok}^{-1} - T_{zr}^{-1})} = 1928 \text{ m}^3.$$

Steady state horizontal winds

5.3: 320 km.

5.4: 8.9 m/s.

5.5: First, we have to calculate the air density for standard atmosphere at 500 mbar (0.7 kg/m^3). The horizontal pressure gradient is 1.15 mbar/100 km.

5.7: It blows under the influence of the Coriolis force ($8.7 \cdot 10^{-4}$ m/s^2), the friction force ($2.3 \cdot 10^{-4}$ m/s^2) and the pressure gradient force ($9.0 \cdot 10^{-4}$ m/s^2). $k = \tan\beta\, f = 10^{-5}$ s^{-1}. $|\nabla p| = 0.45$ mbar/100 km.

5.8: 11.5 m/s.

5.9: $\frac{v_{\text{grad}}}{v_{\text{geo}}} = 0.95.$

5.10: a) $|\nabla p| = 4$ mbar/500 km. $v_g = 6.4$ m/s. b) $v_g = 7.2$ m/s.

5.11: $v_g = 8.3$ m/s, $F_{\text{cor}} = 1.0 \cdot 10^{-3}$ m/s^2, $F_{\text{fr}} = 0.83 \cdot 10^{-3}$ m/s^2, $F_{\text{gra}} = 1.3 \cdot 10^{-3}$ m/s^2.

5.12: $v_g = 17.9$ m/s, $F_{\text{cor}} = 3.1 \cdot 10^{-3}$ m/s^2, $F_{\text{fr}} = 1.7 \cdot 10^{-3}$ m/s^2, $F_{\text{gra}} = 3.6 \cdot 10^{-3}$ m/s^2.

5.13: a) In the tropical hurricane, the centrifugal and the pressure gradient force are equal (we neglect the Coriolis force). $\Delta p = 30.86$ mbar. b) $\Delta p = 30.90$ mbar.

5.14: 961 mbar.

5.15: 602 km.

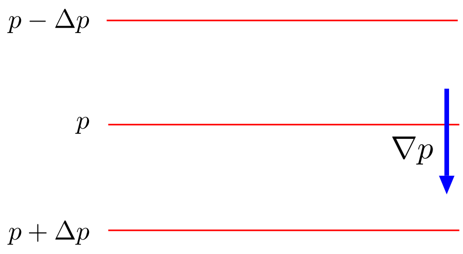
5.16: $p(r) = p_0 + \frac{\rho\Omega}{2} r.$

5.17: The pressure is preserved, if we move from the starting point up along the axis that is tilted for 30 degrees:

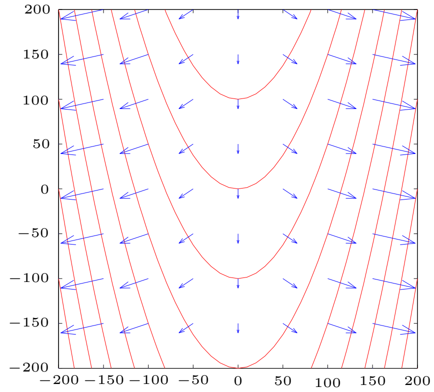
$$\Delta p = \left(\frac{\partial p}{\partial r}\right) R - \rho g \Delta z = 0,$$
$$\left(\frac{\partial p}{\partial r}\right) = \frac{\rho g \Delta z}{R} = \frac{\rho g}{\tan\alpha},$$
$$v = \sqrt{\frac{R}{\rho}\left(\frac{\partial p}{\partial r}\right)} = \sqrt{\frac{Rg}{\tan\alpha}} = 41.2 \text{ m/s}.$$

Local, individual and advective changes

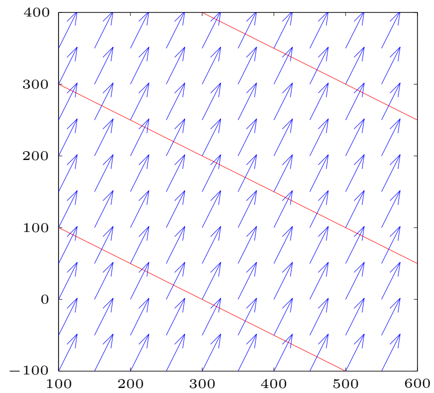
6.2: a) $\nabla p = (0, a)$, b) 996 mbar.



6.3: a) $\nabla p = (2bx, a)$, b) 1001 mbar.



6.4: a) $\nabla p = (b, a)$, b) 1011 mbar.



6.6: a) 0.25 °C/h b) 7.1 m/s.

6.7: 0.25 °C/h.

6.8: 24.4 °C.

6.9: 23.6 °C.

6.10: 1.2 °C/h.

6.11: $\frac{\partial T}{\partial r} = \frac{3\text{ K}}{500\text{ km}}$.

6.12: With the concentrations, we calculate in exactly the same way as with the temperatures. The solution is 2333 s.

Humidity

7.2:

$$f = e/e_s = 0.48.$$

7.3: 284.4 K.

7.4: 255.3 K.

7.5:

$$e(f = 90\%) = 21.3 \text{ mbar}, \quad e(f = 10\%) = 2.4 \text{ mbar},$$
$$\Delta m = m(f = 90\%) - m(f = 10\%) = -8.5 \text{ g}.$$

7.6:

$$m_v = \rho_v \cdot V = \frac{eV}{R_v T} = 0.86 \text{ kg}.$$

7.7: a) 1.242 kg/m^3, b) 1.238 kg/m^3.

7.8: First, we calculate the atmospheric pressure and then the specific humidity, which is $3.5 \cdot 10^{-3}$.

7.9: a) $e_s(T = 30\ °\text{C}) = 43.6$ mbar, $q_s = 0.027$, b) $e_s(T = -15\ °\text{C}) = 1.9$ mbar, $q_s = 0.0012$.

7.10:

$$f = \frac{e}{e_s},$$
$$\frac{df}{f} = \frac{de}{e} - \frac{de_s}{e_s} = \frac{de}{e} - \frac{h_\text{i}}{R_v}\frac{dT}{T^2}.$$

a) Increasing (decreasing) of the temperature for 3 K causes the relative decrease (increase) of the relative humidity by 0.22.

b) Increasing (decreasing) of the vapour pressure for 1% causes the relative increase (decrease) of the relative humidity by 0.01.

7.11: a) 279.9 K.

b) In case of saturation, the absolute humidity is $\rho(T_d) = \frac{e_s(T_d)}{R_v T_d} = 7.7$ g/m^3. The final absolute humidity is $\rho_2 = \frac{e_s(T_2)}{R_v T_2} = 1.5$ g/m^3. The difference of condensed water per unit volume is 6.2 g.

7.12: We assume that the water vapour does not have an effect on the energy balance. The received energy is spent for two things: expansion of the air ($p\Delta V$) and heating of the air ($mc_p\Delta T$). With the use of the gas equation and a little rearrangement, we come to the equation $A = m(c_p + R)\Delta T$. From this equation, we obtain the temperature change $\Delta T = 3.9$ K. To get the relative humidity, we calculate the initial and the new saturated vapour pressure. The new relative humidity is 51%.

7.13: The actual partial pressure of the water vapour, $e = 18.9$ mbar, is higher than the saturated vapour pressure at the new temperature $e_s(T = 20\ °\text{C}) = 15.0$ mbar, so dew will form.

7.14: The amount of the evaporated water per square metre of the soil is proportional to the difference between the actual and the saturated vapour pressure in the layer of the air at the ground:

$$m/S = \frac{\Delta e \cdot h}{R_v T} = 0.17 \text{ kg/m}^2.$$

7.15: In the warm air (T_2), the actual vapour pressure is 11.8 mbar and in the cold air (T_1) 3.8 mbar. From here on, we calculate the absolute humidity of the mixture:

$$\rho(\text{mixture}) = \frac{1}{2}\left(\frac{e_1}{R_v T_1} + \frac{e_2}{R_v T_2}\right),$$

then the partial pressure of the water vapour of the mixture:

$$e(\text{mixture}) = \rho(\text{mixture}) R_v T_2,$$

and finally the new relative humidity:

$$f = 0.34.$$

7.16:

$$m = \frac{\Delta e V}{R_v T} = 0.176 \text{ kg}.$$

7.17: Yes. The saturated vapour pressure after ten hours (8.7 mbar) is lower than the actual vapour pressure (13.7 mbar).

7.18: First, we determine the specific humidity of the mixture which is 0.0100. If the saturation is not reached the final temperature will be the average temperature between 21 °C and 5 °C (because the air masses are mixing in the same proportions), which is 13 °C. Because the saturated vapour pressure at this temperature is lower than the vapour pressure of the mixture, the air will become saturated. We can write the energy equation

$$m c_{\text{pz}}(T - T_{\text{H}}) = m_a h_{\text{i}} + m c_{\text{pz}}(T_{\text{T}} - T),$$

where the indexes H and T are the temperature of the warm and the cold air and m_a the mass of the water that is condensed. We can write this as the difference of the masses of the water vapour before and after the condensation

$$m_a = m_{v1} - m_{v2} = mq - mq_s(T) = mq - \frac{mR}{pR_v}\, e_s(T),$$

where q is the specific humidity of the mixture. From there we obtain

$$c_{\text{pz}}(T - T_{\text{H}}) = q h_{\text{i}} - \frac{R h_{\text{i}}}{p R_v} e_s(T) + c_{\text{pz}}(T_{\text{T}} - T).$$

Because we cannot analytically express the temperature from this equation ($e_s(T)$ appears in the exponent), we need to solve the equation numerically. The result is 286.4 K. From here, we determine that $m_a = 0.35$ g of the water condensed per kilogram of the air.

7.19: The saturated vapour pressures are 12.3 mbar and 11.5 mbar. From every cubic metre 0.59 g of water is eliminated.

7.20: The vapour pressures do not change since the specific humidity of the water vapour in the air does not change. Since the atmospheric pressure is also constant, the vapour pressure does not change.

7.21: Above the water, the saturated vapour pressure is 2.4 mbar. Above the ice, we use the same Claussius-Clapeyron equation in which the specific latent heat of evaporation h_{i} is replaced with the specific heat of sublimation h_{s}. The result is 2.2 mbar.

7.22:

$$\frac{f_{\text{water}}}{f_{\text{ice}}} = \frac{e_{s,\text{ice}}}{e_{s,\text{water}}} = 1.05.$$

7.23: We want to write the temperature profile with the sinusoidal function of time (t). Therefore

$$T(t) = T_A + T' \cdot \sin[a \cdot t + b],$$

where $T_A = 288$ K and $T' = 10$ K. We have to determine the constants a and b so that the temperature at 14:00 will be maximal and at 8:00 minimal. This is completed when the argument in sinus is at 14:00 equal $\pi/2$ and at 8:00 $-\pi/2$. Therefore, for a, b we obtain the system of the two equations, which yield $a = \frac{\pi}{6}$ h^{-1} and $b = -\frac{11\pi}{6}$.

The relative humidity is dependent on the temperature:

$$f = \frac{e}{e_s(T)} = \frac{qpR_v}{R}\frac{1}{e_s(T)} = \frac{qpR_v}{Re_{s0}}\,\mathrm{e}^{-\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T}\right)}$$
$$= \frac{qpR_v}{Re_{s0}}\,\mathrm{e}^{-\frac{h_\mathrm{i}}{R_v}\left(\frac{1}{T_0}-\frac{1}{T_A+T'\cdot\sin[a\cdot t+b]}\right)}.$$

At 10:00, the relative humidity will be 40%.

7.24: The saturated vapour pressures at the temperature of the dry and the wet bulb thermometers are $e_s(T) = 14.08$ mbar and $e_s(T_m) = 12.13$ mbar. We calculate the vapour pressure with the psychometric equation:

$$e = e_s(T_m) - \frac{c_{\mathrm{pz}}pR_v}{h_\mathrm{i}R}(T - T_m) = 11.1 \text{ mbar}.$$

The relative humidity is 79%.

7.25: From the psychometric equation, we calculate the partial pressure of the water vapour 8.8 mbar. The dew point temperature is 278.1 K.

7.26: With the use of the psychometric equation, we arrive at the following equation:

$$f \cdot e_s(T) = e_s(T_m) - \frac{c_{\mathrm{pz}}pR_v}{h_\mathrm{i}R}(T - T_m).$$

The equation is implicit, because we cannot express T_m analytically. The numerical solution is $T_m = 14$ °C.

7.27: a) We assume that all drops that fall on the bridge, immediately cool down, and freeze to the temperature of the bridge. The energy that is released at the cooling and freezing of the water drops and the cooling of the frozen drops is used for the heating of the bridge. We can write the energy used for heating of the bridge as $m_M c_{pM} \Delta T' = \rho_M S d c_{pM} \Delta T'$, where S, d are the surface area and the thickness of the bridge and $\Delta T'$ is 1 K.

The energy that is released by the cooling of the drops can be similarly written as $\rho_a SRRtc_{\mathrm{pa}}\Delta T''$ where RR is the intensity of the rainfall (1 mm/h) and t is time. Similar is true for the cooling of the frozen drops by one degree, except that instead of c_{pa} we use c_{pl}. The energy, that is used for the freezing of the drops can be written as $\rho_v SRRth_\mathrm{t}$. The result is 0.2 h.

b) We write the energy for warming up the bridge similarly as before. The warming of the bridge is caused only by the cooling of the water droplets (there is no more freezing). The result is 3.8 h.

7.28: The total rainfall accumulated in the rain gauge is independent of the horizontal wind (if the rain gauge is placed horizontally). It is dependent only on the vertical velocity of the droplets. In three hours, the mass is:

$$m = \rho_k \Delta t w S_0 = 7.8 \text{ kg}.$$

If the snowflakes are falling, the accumulated mass is 2.2 kg.

7.29:

$$T_v = T(1 + \frac{R_v - R}{R} q) = 306.7 \text{ K}.$$

Adiabatic changes

8.3: We calculate the dew point temperature at the ground. Starting from the equation for the specific humidity $q = \frac{e}{p}\frac{R}{R_v}$, we calculate the vapour pressure $e = 176.7$ Pa, which determines the dew point temperature $T_d = -16$ °C. The air first rises from the ground along the dry adiabat, until at the height of $z_B = 3.72$ km the air becomes saturated. Then it is rising along the moist adiabat to the final height of 6 km, where it has the temperature $T = -38.2$ °C.

8.4: From the height of the cloud base, we calculate the dew point temperature of the air at the ground: $T_d = 4.6$ °C. The relative humidity is the ratio between the saturated pressure of the water vapour at the dew point temperature ($e_s(4.6\ °C) = 848.1$ Pa) and the saturated pressure of the water vapour at the temperature 15 °C, ($e_s(15\ °C) = 1704.1$ Pa); therefore, $f = 50\%$.

8.5: The height of the slope is $h = vt \cdot \sin\alpha = 870$ m. a) $\Delta T = -\Gamma_a h - \left(\frac{\partial T}{\partial z}\right) h = -4.4$ K. b) $\Delta T = -\Gamma_s h - \left(\frac{\partial T}{\partial z}\right) h = 0.9$ K.

8.6: The dew point temperature at the ground is $T_d = 11.6$ °C. The height of the cloud base is $h = 0.4$ km, and above the air is saturated. The relative humidity at 500 m is therefore 100%.

8.7: When rising, the air temperature decreases to $T_2 = 7$ °C. The partial pressure of the water vapour before the rising is $e_1 = \rho_{v1} R T_1 = 582.6$ Pa; at the height of 1000 m, the partial pressure of water vapour and the absolute humidity are:

$$e_2 = e_1 \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 515.3 \text{ Pa}$$
$$\rho_{v2} = \frac{e_2}{RT_2} = 6.4 \text{ g/m}^3$$

8.8: From the specific humidity, we calculate the partial pressure of the water vapour at the ground $e_1 = 803$ Pa and then the relative humidity of the air at the ground $f_1 = 33.9\%$. At the height of 500 m, the rising air has the temperature of 15 °C, the partial pressure of the water vapour during the rising is changing depending on the temperature:

$$e_2 = e_1 \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 756 \text{ Pa}$$

The relative humidity at the height of 500 m is $f_2 = e_2/e_{s2} = 44.0\%$. The level of the condensation is at the height of 1.94 km.

8.9: a) We calculate the air temperature (T_2) and the saturated vapour pressure (e_2) at that temperature. Then we calculate the partial pressure of the water vapour at the height of 1000 mbar and the final humidity f_1 at the ground:

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{R}{c_p}} = 287.3 \text{ K}$$
$$e_2 = e_s(T_2) = 1628.75 \text{ Pa}$$
$$e_1 = e_2 \left(\frac{T_1}{T_2}\right)^{\frac{c_p}{R}} = 1917.0 \text{ Pa}$$
$$f_1 = \frac{e_1}{e_s(T_1)} = 50.7\%$$

b) The changes are small and happening fast; we can use the differential form of the energy equation:

$$mc_p\Delta T - V\Delta p + h_\mathrm{i}\Delta m_v = 0$$
$$c_p\Delta T - \frac{\Delta p}{\rho} + h_\mathrm{i}\Delta q = 0$$

The quantity of the condensed water vapour that is expressed as the change of the specific humidity:

$$\Delta q = \frac{\Delta p}{\rho h_\mathrm{i}} - \frac{c_p}{h_\mathrm{i}}\Delta T = 0.3 \text{ g/kg}$$

8.10: $z = 1.4$ km

8.11: The height, at which the cumulus clouds will occur (if the convection extends to this height) is $z = 2$ km. In reality, the free convection extends only to the height of 1.1 km, therefore clouds did not form.

8.12: $\rho_1 = 1.2$ kg/m^3, $T_2 = 255.6$ K, $\rho = 0.95$ kg/m^3.

8.13: a) The dew point temperature at the ground is $T_d = 9.6$ °C. The cloud base is at the height of $z_B = 648$ m. Yes, the hill has orographic clouds. b) $T_d = 13.4$ °C, $z_B = 200$ m.

8.14: a) The air is mixed to the height of 1 km. b) No, cloudiness did not appear.

8.15: a) $T_d = 14.1$ °C.

b) The condensation level is at the height of $z = 846$ m, where the air temperature is $T = T_0 - \Gamma_a\Delta z = 12.5$ °C, and the atmospheric pressure is

$$p_2 = p_1 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = 903 \text{ mbar}$$

8.16: The dew point temperature at the ground is $T_d = 15.45$ °C. The condensation occurs at the height of $z_1 = 300$ m above the sea level. $z = 700$ m above the sea the relative humidity is 100% and the air temperature $T = T_0 - \Gamma_a z_1 - \Gamma_s(z - z_1) = 12.2$ °C.

8.17: Yes. Solve with the help of the rotated $(T, \ln p)$ diagram.

8.18: a) $e = 1274.8$ Pa, $T_d = 10.6$ °C.

b) Solve with the help with the rotated $(T, \ln p)$ diagram. $T = 18$ °C.

8.19: Calculate the partial pressure of the water vapour on both sides of the hill. We assume that the atmospheric pressure is 1000 mbar and then calculate the specific humidity on both sides. From the air $\Delta q = 4$ g/kg of precipitation is removed.

8.20: The air mixes to the height of 3.6 km. We can solve the example by calculation or with the help of (T, z) diagram.

8.21:

$$\omega = \sqrt{g\frac{\Gamma_a + \left(\frac{\partial T}{\partial z}\right)}{T_{ok}}} = 0.01 \text{ s}^{-1}$$

In a non-stable atmosphere, it does not come to the fluctuations, but the air displaced from the equilibrium position keeps on accelerating.

8.22: We assume that the vertical speed of the wind $w = v \cdot \sin\alpha = 3.4$ m/s is not changing with the height. The maximum amount of the precipitation is equal to the total amount of the condensed water vapour:

$$\begin{aligned} RR &= \int_{\text{base}}^{\text{top}} \frac{c_p}{h_\text{i}}(\Gamma_a - \Gamma_s(z))\, w(z)\rho(z)\,\mathrm{d}z \\ &= \frac{c_p}{gh_\text{i}} \int_{1000\text{ mbar}}^{500\text{ mbar}} (\Gamma_a - \Gamma_s(p))\, w(p)\,\mathrm{d}p \\ &\doteq \frac{wc_p}{gh_\text{i}} \sum_i (\Gamma_a - \Gamma_s(p))\Delta p_i \\ &\doteq 347.5\text{ mm} \end{aligned}$$

Emagrams

9.2: The solution is not drawn, b) the example is solved similarly as the examples from the caption about the humidity: 52% (1000 mbar) and 40% (700 mbar), c) 200 mbar, d) Nowhere. The dew point temperature is nowhere equal to the air temperature e) around 890 mbar, f) around 890 mbar, g) around 770 mbar, h) around 170 mbar. i) The ground warms up during the day when the Sun shines on them. At the same time as the ground warms up, the air at the ground also warms up, while the air temperature higher up does not change. This warming at the ground can cause the air at the ground to become warmer than the surroundings (the convection begins). In this case, forced rising is not needed. Meanwhile, the amount of water vapour in the air near the ground stays constant (assuming there is no evaporation from the ground). That means that the dew point temperature near the ground does not change. The problem seeks to determine by how much the air at the ground will need to warm up. It needs to be warmer than the surrounding air when it reaches the lifting condensation level. This is determined by following the curve of the dew point temperature to the height at which it crosses the temperature of the surrounding air (around the height of 750 mbar). The lifted condensation level has to be at this height (or higher). From this height, we simply follow the dry adaiabat line downwards to the ground. Where the line crosses the ground, we can read the temperature (around 43 °C). The air at the ground should thus have warmed up by 9 °C (from 34 °C to 43 °C).

9.3: a) decreases from 12 °C (800 mbar) to −20 °C (500 mbar), therefore the temperature decreases by approximately 32 °C, b) Two cloudy layers: 500–400 mbar and 300–250 mbar, c) 250 mbar, d) around 850 mbar, e) around 610 mbar, f) by approximately 12 °C (to approximately 32 °C), g) to the height around 240 mbar.

9.4: a) around 5 °C (700 mbar) and −40 °C (500 mbar), b) There are no cloudy layers, c) 900 mbar, d) No. For free convection, the hill needs to reach 800 mbar, e) to the top of the hill (850 mbar), f) for approximately 6 °C (to approximately 31 °C), g) to the height around 200 mbar.

9.5: a) around −23 °C, b) One cloudy layer is around 700 mbar, c) 930 mbar, d) the free convection almost does not occur. There is just a small region of free convection below the height of 700 mbar, e) The example is a bit reversed from the previous cases, but the logic is similar. From the temperature of the surrounding air at the height of 200 mbar, we descend along the moist adiabat line to where it intersects the line, which describes the change of the dew point temperature. At this height (around 580 mbar), the lifted condensation level is located. From this height, you follow the dry adiabat line downward to the ground. In our case, the line intersects the ground around the temperature of 50 °C. The air at the ground has to warm up by 35 °C (from 15 °C to 50 °C), f) the cloud will extend from the lifted condensation level (around 580 mbar) to the height of 200 mbar.

Radiation

10.2: $P = \varepsilon\sigma S(T^4_{\mathrm{wals}} - T^4_{\mathrm{skin}}) = 114$ J/s.

10.3: $P = \varepsilon\sigma T^4 S = 286.9$ W.

10.4: The energy, that the Earth receives from the Sun in one day is: $E_s = j_0 t \cdot \pi R_z^2 \cdot (1-a)$. The energy that is consumed by the evaporation: $E_i = mq_i = \rho_v h q_i \cdot 4\pi R_z^2$. The energy portion, that is used for the evaporation: $E_i/E_s = 2 \cdot 10^{-4} = 0.03\%$.

10.5: We assume a linear profile of the Sun's elevation during the day and obtain a sinusoidal changing of the flux density. The energy the area receives in one day is:

$$E_d = \int_0^{t_0} j_0 S \sin\left(\frac{\pi}{3}\frac{t}{t_0}\right) \mathrm{d}t + \int_{t_0}^{2t_0} j_0 S \sin\left(\frac{2\pi}{3} - \frac{\pi}{3}\frac{t}{t_0}\right) \mathrm{d}t = j_0 S \frac{3t_0}{\pi},$$

where t_0 is 6 h. For the evaporation of $h = 1200$ mm of precipitation the energy of $E_i = qm_v = qSh\rho_v$ is needed. The number of days that are needed for the evaporation of all water is: $N = E_i/E_d = 106$.

10.6: In a radiative equilibrium, the Earth is receiving the same amount of power of solar energy that it is emitting:

$$(1-a)j_0 \cdot \pi R_z^2 = ((1-\varepsilon_{\mathrm{atm}})j_{\mathrm{ground}} + j_{\mathrm{atm}}) \cdot 4\pi R_z^2$$

We consider $j_{\mathrm{ground}} = \varepsilon_{\mathrm{ground}}\sigma T^4_{\mathrm{ground}}$ and $j_{\mathrm{atm}} = \varepsilon_{\mathrm{atm}}\sigma T^4_{\mathrm{atm}}$ and calculate the temperature of the atmosphere:

$$T_{\mathrm{atm}} = \sqrt[4]{\frac{\frac{j_0(1-a)}{4} - (1-\varepsilon_{\mathrm{atm}})\sigma T^4_{\mathrm{ground}}}{\varepsilon_{\mathrm{atm}}\sigma}} = 226.8 \text{ K} = -46\ ^\circ\text{C}.$$

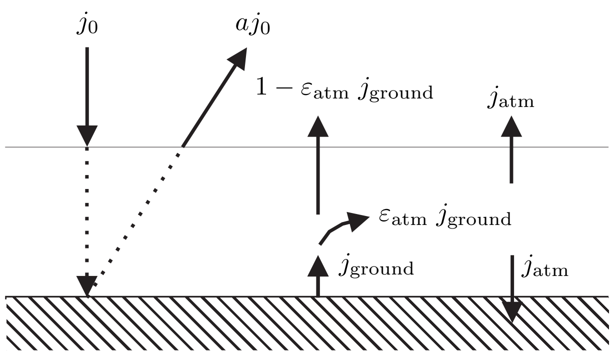


Figure 13.1: The illustration to the solution of the 6th problem.

10.7: a) All received power is used for heating the lake: $P_e t = mc_v \Delta T$. We can calculate that $\frac{\Delta T}{t} = \frac{P_e}{mc_v} = 0.01$ K/day.

b) When the new equilibrium is established, the power from the power plant is equal to the power by radiation: $\varepsilon T_{\text{new}}^4 S = \varepsilon T^4 S + P_e$. The new equilibrium temperature:

$$T_{\text{new}} = \sqrt[4]{T^4 + \frac{P_e}{\varepsilon\sigma S}} = 284 \text{ K}.$$

The lake warms up by 1 K.

c) $\frac{\Delta m}{q_i} = \frac{P_e}{q_i} = 0.17$ mm/day.

10.8: From the equation for the energy equilibrium: $E = m_v q_i + \varepsilon\sigma T^4 S t$, we calculate the temperature $T = 12.7$ °C.

10.9: a) 53.5 days, b) 90.5 days.

10.10: a) 53.5 days, b) 0.20 m, c) 123 days.

10.11: a) 725 W, b) 774 W.

10.12: We solve the diffusion equation:

$$\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial t^2} = \frac{\partial T}{\partial t}$$

with the initial condition $T = T_0 + (T_{max} - T_0)\sin((t - t_0)\frac{\pi}{12\text{ h}})$. We get the equation, that describes the changing of the temperature with the time at the different heights: $T = T_0 + (T_{max} - T_0)\,\mathrm{e}^{-k_1 z}\sin((t - t_0)\frac{\pi}{12\text{ h}} k_1 z)$, where $k_1 = \pi/(2D \cdot 12\text{ h})$ where $D = \lambda/\rho c_p$. At the depth of 15 cm the amplitude of the temperature fluctuation is 8 K.

10.13: $T_{\text{ground}} = 278$ K.

10.14: For the ground and for every atmosphere layer the sum of the energy fluxes is 0.

$$\begin{aligned}
&\text{ground:} && j_{\text{ground}} = (1-a)j_s + j_1 + tj_2 + tj_3\\
&\text{1. layer:} && 2j_1 = (1-t)j_{\text{ground}} + (1-t)j_2\\
&\text{2. layer:} && 2j_2 = (1-t)j_1 + (1-t)j_3\\
&\text{3. layer:} && 2j_3 = (1-t)j_2
\end{aligned}$$

The ground temperature is 300 K.

10.15: a) We assume that the elevation of the sun between 11:30 and 12:30 is not changing.

$$\begin{aligned}
Q &= j_0 S_0 \cos\varphi_0 t\\
&= 1480 \text{ J}
\end{aligned}$$

b) We calculate the equilibrium temperature at noon using the radiation balance for the black body, where we assume that the pad radiates only upwards:

$$T = \sqrt[4]{\frac{j_0\cos\varphi_0}{\sigma}} = 331.5 \text{ K}.$$

c) We describe the change of the zenith angle of the Sun with the time from the sunrise ($t = 0$) to the sunset ($t = t_0$) with the equation:

$$\cos\varphi(t) = \cos\varphi_0 \cdot \frac{1}{2}\left(1 + \sin\frac{2\pi}{t_0}\left(t - \frac{t_0}{4}\right)\right), \quad t \in [0, t_0].$$

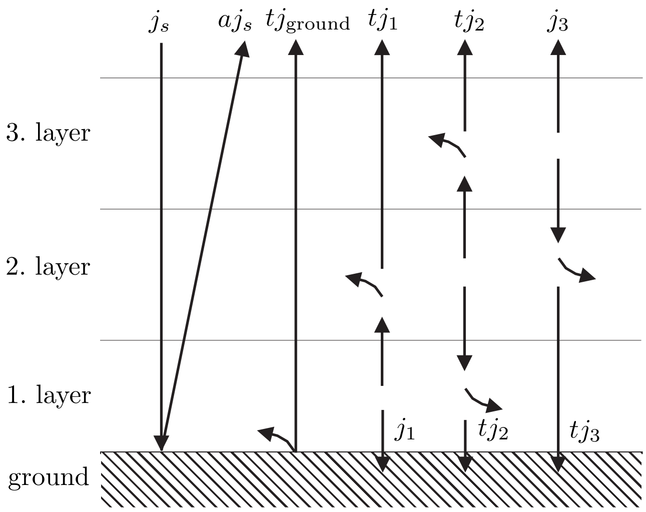


Figure 13.2: The illustration to the solution of the 13th problem.

The total received energy is:

$$\begin{aligned} Q_s &= j_0 S_0 \int_0^{t_0} \cos\varphi(t)\,\mathrm{d}t \\ &= j_0 S_0 \cos\varphi_0 \frac{t_0}{2} \\ &= 9247\ \mathrm{J} \end{aligned}$$

10.16: We assume that the thermometer bulb is spherical. The received power of the solar radiation and the radiation of the surrounding air is equal to the radiated power of the bulb:

$$\begin{aligned} (1-a_t) j_0 S_0 + \varepsilon_z \sigma T_z^4 4 S_0 &= \varepsilon_t \sigma T_{ter}^4 4 S_0 \\ a_t &= 1 - \frac{4\sigma(\varepsilon_t T_{ter}^4 - \varepsilon_z T_z^4)}{j_0} \\ &= 0.04 \end{aligned}$$

Fronts

11.2: The movement speed of the front is defined by the wind component, which is perpendicular to the front. This component has to be the same in the warm and cold sectors. If β is the angle between the front and the wind vector in the warm sector, the equation: $v_t \sin\beta = v_h \sin(\pi/4 + \pi/6 - \beta)$ has to be true. We calculate the angle $\beta = 29.2°$. The speed of the front is then $u = v_t \sin\beta = 12.5$ m/s.

11.3: From the tilt of the front, we calculate the wind shear at the front and the wind in the warm sector. The latter blows at the speed of 35 m/s from the direction of 129.2°.

11.4: The front orientation is SW-NE and travels at the speed of 7.1 m/s.

11.5: Speed: 14.9 m/s, direction: 294.1°.

11.6: a) The speed of the wind in the cold air is 26 m/s. The front tilt is $\tan\alpha = 1/73$. b) The speed of the front is 10 m/s, we can expect the wind change to occur after 2 hours.

11.7: We use the equations that are also valid for the front. The tilt of the upper limit of the cold-air lake is $\tan\alpha = 1/231$. The inversion level on the other side of the basin is 273 m, which is at the southern part of the basin.

11.8: $\tan\alpha = 1/456$.

11.9: The tilt of the front is $\tan\alpha = 1/159$. After the winds in the warm air calm down, the tilt of the surface will be $\tan\alpha = 1/484$.

11.10: The tilt of the front is 1/326. The cirrus cloud appears approximately 2600 km before the front. From the 1000-m high hill, we can see the cirrus clouds that at approximately 400 km away (we take into account that the radius of the Earth is $R_z = 6378$ km).

TAF and METAR weather reports

12.2: The measurement that was performed at 9:20 UTC reports, that at the EGLL (Heathrow) airport the wind was measured from the direction of 260° at the speed of 5 knots. The visibility exceeds 10 km, there are no clouds below the height of 5000 feet and there is no fog and no precipitation. The temperature is 15 °C and the dew point temperature 14 °C. The pressure, reduced to the sea level, is 1013 mbar. In the next two hours, no significant changes in the weather are expected.

12.3: The measurement that was performed at 15:50 UTC reports, that at the EDDL (Dusseldorf) airport the wind was measured from the direction of 260° at the speed of 5 knots. The general visibility is 550 m; the visibility along landing runway 23L is 450 m. Freezing fog is present. Cloudiness is 8/8 at the height of 200 feet. The temperature and the dew point temperature are −2 °C. The pressure, reduced to the sea level, is 994 mbar. The forecast is that in the next two hours cloudiness will rise to the height of 500 feet.

12.4: The measurement that was performed at 9:00 UTC reports, that at the EIDW (Dublin) airport the wind was measured from the direction of 240° at the speed of 35 knots and with gusts up to the speed of 55 knots. The wind direction is changing in the interval from 210° to 270°. The visibility is 1700 m. There are intensive showers with rain. Cloudiness from 5/8 to 7/8 is at the height of 700 feet. At the height of 1500 feet, 8/8 cloudiness is located, where the base of the cumulonimbus is located. The temperature is 8 °C, and the dew point temperature 7 °C. The forecast is that the visibility will increase to 3500 m for a shorter period.

12.6: The forecast was given on the 4^{th} of the month at 06:00 UTC for the LJLJ (Ljubljana) airport. The forecast is valid for a period from 4^{th} at 07:00 UTC to the next day at 16:00 UTC. There will be variable wind at the speed of 3 knots, visibility 1500 m and light snow. Cloudiness from 1/8 to 2/8 will be at the height of 100 feet. Cloudiness from 5/8 to 7/8 will be at the height of 500 feet. Cloudiness of 8/8 will be at the height of 2000 feet. From 07:00 UTC to 16:00 UTC, the visibility will decrease to 800 m. The intensity of snow fall will increase.

12.7: The forecast was given on the 11^{th} of the month at 05:30 UTC for the LJMA (Maribor) airport. The forecast is valid for a period from 4^{th} of the month at 07:00 UTC to the next day at 16:00 UTC. There will be variable wind at a speed of 1 knot, visibility 3000 m and light rain with snow. The cloudiness of 8/8 is at the height of 3300 feet.

In the time between 07:00 UTC and 09:00 UTC, the circumstances will change. The visibility will be 1500 m, only snow will be falling, at the height of 2000 feet the cloudiness from 5/8 to 7/8 will appear. Cloudiness of 8/8 will stay at the height of 3300 feet.

From 07:00 UTC to 16:00 UTC, the visibility will be temporarily reduced to 800 m. The intensity of snow fall will increase.

In the time between 12:00 UTC and 15:00 UTC, the wind will change to 20° and 10 knots.

Chapter 14

A list of used symbols and codes

c_{pa}	specific heat of water at constant pressure, $c_{\text{pa}} = 4181\ \text{J/kg K}$
c_{pl}	specific heat of ice at constant pressure, $c_{\text{pl}} = 2114\ \text{J/kg K}$
c_{pv}	specific heat of water vapour at constant pressure, $c_{\text{pv}} = 1847\ \text{J/kg K}$
c_{pz}	specific heat of dry air at constant pressure, $c_{\text{pz}} = 1004\ \text{J/kg K}$
g_0	standard value of gravitational acceleration, $g_0 = 9.81\ \text{m/s}^2$
h_{i}	specific latent heat of water evaporation, $h_{\text{i}} = 2.50\ \text{MJ/kg}$
h_{s}	specific latent heat of water sublimation, $h_{\text{s}} = 2.83\ \text{MJ/kg}$
h_{t}	specific heat of fusion, $h_{\text{t}} = 0.33\ \text{MJ/kg}$
j_0	solar constant, energy flux density of solar radiation at the top of the Earth's atmosphere, $j_0 \approx 1400\ \text{W/m}^2$
R^*	specific gas constant, $R^* = 8317\ \text{J/kmol K}$
R	specific gas constant for air, $R = 287\ \text{J/kg K}$
R_v	specific gas constant for water vapour, $R_v = 461.5\ \text{J/kg K}$
Γ_a	(negative) individual change of temperature at adiabatic displacement of unsaturated air in vertical direction, $\Gamma_a = -\frac{\text{d}T}{\text{d}z} = 10\ \text{K/km}$
Γ_s	(negative) individual change of temperature at adiabatic displacement of saturated air in vertical direction, $\Gamma_s = -\frac{\text{d}T}{\text{d}z}$

Chapter 15

Appendix

$T[^\circ C]$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
−20	125.74	124.66	123.59	122.53	121.47	120.43	119.39	118.36	117.34	116.32
−19	137.00	135.83	134.68	133.53	132.39	131.26	130.14	129.03	127.92	126.83
−18	149.15	147.89	146.65	145.41	144.18	142.96	141.75	140.55	139.36	138.17
−17	162.26	160.90	159.56	158.22	156.90	155.58	154.28	152.98	151.69	150.42
−16	176.39	174.93	173.48	172.04	170.61	169.19	167.79	166.39	165.00	163.63
−15	191.61	190.04	188.48	186.93	185.39	183.86	182.34	180.84	179.34	177.86
−14	208.00	206.30	204.62	202.95	201.30	199.65	198.02	196.40	194.79	193.20
−13	225.62	223.80	221.99	220.20	218.42	216.65	214.89	213.15	211.42	209.70
−12	244.57	242.61	240.67	238.74	236.82	234.92	233.03	231.16	229.30	227.45
−11	264.92	262.82	260.73	258.66	256.60	254.56	252.53	250.52	248.52	246.54
−10	286.77	284.51	282.28	280.05	277.84	275.65	273.47	271.31	269.17	267.04
−9	310.21	307.79	305.39	303.01	300.64	298.29	295.95	293.63	291.33	289.04
−8	335.35	332.76	330.18	327.62	325.08	322.56	320.06	317.57	315.10	312.65
−7	362.28	359.50	356.75	354.01	351.29	348.58	345.90	343.23	340.59	337.96
−6	391.12	388.15	385.20	382.26	379.35	376.46	373.58	370.73	367.89	365.08
−5	421.99	418.81	415.65	412.51	409.39	406.30	403.22	400.17	397.13	394.12
−4	455.01	451.60	448.22	444.87	441.53	438.22	434.93	431.66	428.42	425.19
−3	490.30	486.66	483.05	479.46	475.90	472.36	468.84	465.35	461.88	458.43
−2	528.00	524.11	520.26	516.42	512.62	508.84	505.08	501.35	497.64	493.95
−1	568.25	564.10	559.99	555.90	551.83	547.79	543.78	539.80	535.84	531.90
−0	611.20	606.78	602.39	598.02	593.69	589.38	585.10	580.84	576.62	572.42
0	611.20	615.65	620.13	624.64	629.17	633.74	638.33	642.96	647.61	652.29
1	657.01	661.75	666.52	671.33	676.16	681.03	685.93	690.86	695.82	700.81
2	705.83	710.89	715.97	721.09	726.24	731.43	736.64	741.89	747.18	752.49
3	757.84	763.23	768.64	774.09	779.58	785.10	790.65	796.24	801.87	807.52
4	813.22	818.95	824.71	830.52	836.35	842.23	848.14	854.08	860.07	866.09
5	872.15	878.24	884.38	890.55	896.76	903.00	909.29	915.61	921.98	928.38
6	934.82	941.30	947.82	954.38	960.98	967.62	974.31	981.03	987.79	994.60
7	1001.44	1008.33	1015.26	1022.23	1029.24	1036.30	1043.40	1050.54	1057.73	1064.95
8	1072.23	1079.54	1086.90	1094.30	1101.75	1109.25	1116.78	1124.37	1132.00	1139.67
9	1147.39	1155.16	1162.97	1170.83	1178.74	1186.69	1194.69	1202.74	1210.83	1218.98
10	1227.17	1235.41	1243.70	1252.04	1260.43	1268.86	1277.35	1285.89	1294.48	1303.11
11	1311.80	1320.54	1329.33	1338.18	1347.07	1356.02	1365.02	1374.07	1383.17	1392.33
12	1401.54	1410.80	1420.12	1429.49	1438.92	1448.40	1457.94	1467.53	1477.18	1486.88
13	1496.64	1506.46	1516.33	1526.26	1536.25	1546.29	1556.39	1566.55	1576.77	1587.05
14	1597.39	1607.78	1618.23	1628.75	1639.32	1649.96	1660.65	1671.41	1682.23	1693.11
15	1704.05	1715.05	1726.12	1737.25	1748.44	1759.69	1771.01	1782.40	1793.84	1805.35
16	1816.93	1828.57	1840.28	1852.05	1863.89	1875.80	1887.77	1899.81	1911.92	1924.09
17	1936.34	1948.65	1961.03	1973.48	1986.00	1998.58	2011.24	2023.97	2036.77	2049.64
18	2062.58	2075.60	2088.68	2101.84	2115.07	2128.38	2141.75	2155.21	2168.73	2182.33
19	2196.01	2209.76	2223.58	2237.48	2251.46	2265.51	2279.65	2293.85	2308.14	2322.50
20	2336.95	2351.47	2366.07	2380.75	2395.51	2410.35	2425.27	2440.27	2455.35	2470.52
21	2485.76	2501.09	2516.51	2532.00	2547.58	2563.24	2578.99	2594.82	2610.74	2626.74
22	2642.83	2659.00	2675.26	2691.61	2708.05	2724.57	2741.19	2757.89	2774.68	2791.56
23	2808.53	2825.59	2842.74	2859.98	2877.31	2894.74	2912.25	2929.86	2947.57	2965.36
24	2983.25	3001.24	3019.32	3037.50	3055.77	3074.13	3092.60	3111.16	3129.82	3148.57
25	3167.43	3186.38	3205.44	3224.59	3243.84	3263.19	3282.65	3302.20	3321.86	3341.62
26	3361.48	3381.45	3401.51	3421.69	3441.96	3462.35	3482.84	3503.43	3524.13	3544.94
27	3565.85	3586.87	3608.00	3629.24	3650.59	3672.05	3693.62	3715.30	3737.09	3758.99
28	3781.00	3803.13	3825.37	3847.72	3870.19	3892.77	3915.47	3938.28	3961.21	3984.25
29	4007.41	4030.69	4054.09	4077.61	4101.24	4124.99	4148.87	4172.86	4196.98	4221.22
30	4245.58	4270.06	4294.66	4319.39	4344.24	4369.22	4394.33	4419.55	4444.91	4470.39

Table of the saturated vapour pressure over water in Pa. The following equation was used:

$$e_s(T) = 611.2\ \text{Pa} \cdot \exp\left(\frac{17.67 \cdot T}{T + 243.5}\right),$$

where the temperature T is in °C. Example: $e_s(-3.2\ °\text{C}) = 483.05$ Pa, $e_s(3.2\ °\text{C}) = 768.64$ Pa
Literature: **Rogers, R.R. in Yau, M.K.**: *A Short Course in Cloud Physics*, Pergamon Press, page 16.

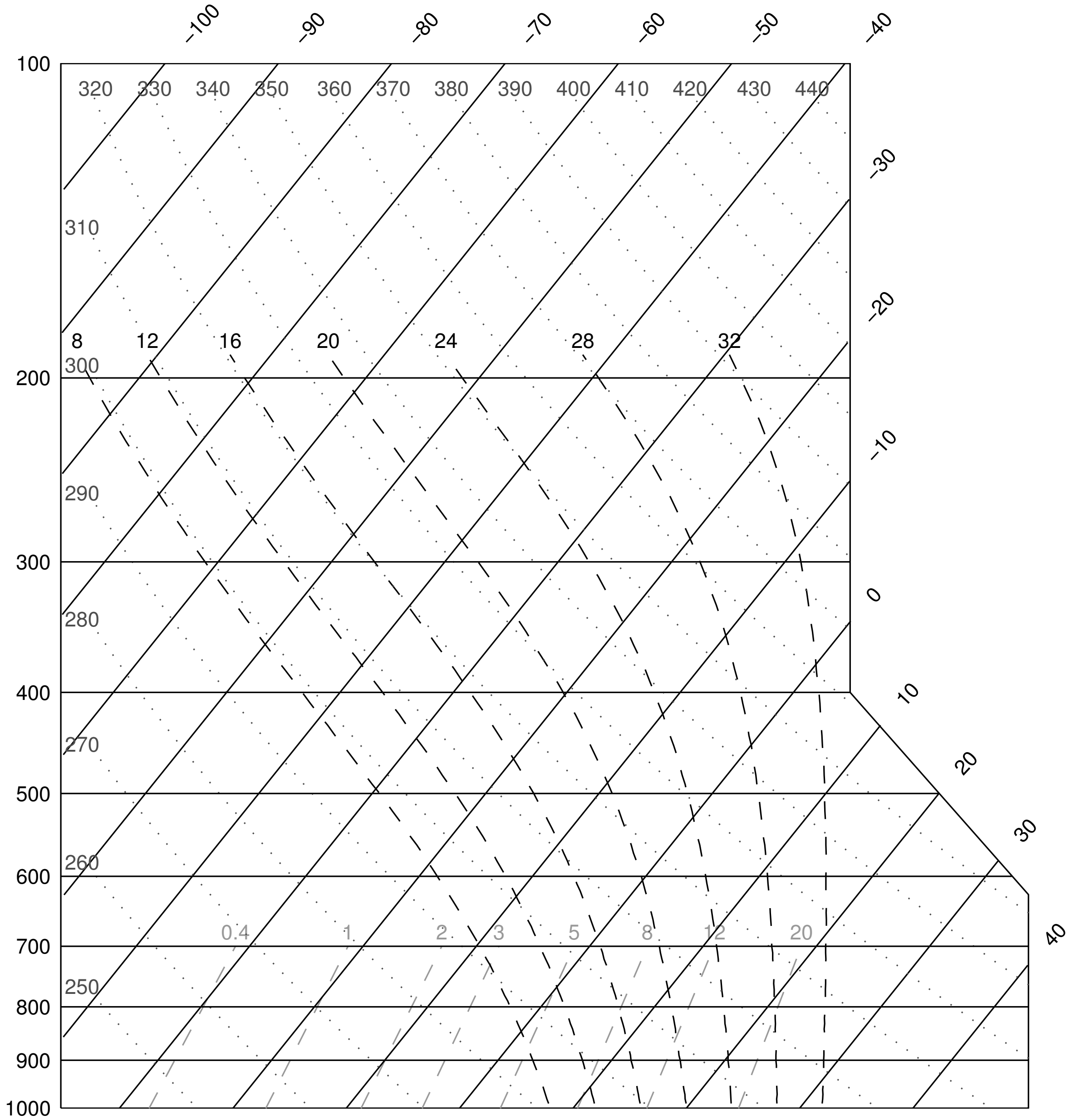


Figure 15.1: Empty Skew-T Log-P emagram.

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Saša Gaberšek, PhD, is a researcher at U.S. Naval Research Laboratory, USA.

Gregor Skok is asis. prof. of meteorology at Faculty of Mathematics and Physics, University of Ljubljana.

Rahela Žabkar, PhD, is an air quality expert at Slovenian Environment Agency.

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